

4.17 You can infer from Problem 4.3 that $1 \leftrightarrow \delta(\mu, \nu)$ and $\delta(t, z) \leftrightarrow 1$. Use the first of these properties and the translation property in Table 4.3 to show that the Fourier transform of the continuous function $f(t, z) = \sin(2\pi\mu_0 t + 2\pi\nu_0 z)$ (there is a typo in the book, which shows a sinusoidal of amplitude A) is

$$F(\mu, \nu) = (j/2)[\delta(\mu + \mu_0, \nu + \nu_0) - \delta(\mu - \mu_0, \nu - \nu_0)]$$

4.21 The need for image padding when filtering in the frequency domain was discussed in Section 4.6.6. We showed in that section that images needed to be padded by appending zeros to the ends or rows and columns in the image (see the following image on the left). Do you think it would make a difference if we centered the image in and surrounded it by a border of zeros instead (see image on the right), but without changing the total number of zeros used? Explain.



4.23 You know from Table 4.2 that the dc term, $F(0,0)$, of a DFT is proportional to the average value of its corresponding spatial image. Assume that the image is of size $M \times N$. Suppose that you pad the image with zeros to size $P \times Q$, where P and Q are given in Eqs. (4.6-31) and Eqs. (4.6-32). Let $F_p(0,0)$ denote the dc term of the DFT of the padded function.

- What is the ratio of the average values of the original and padded images?
- Is $F_p(0,0) = F(0,0)$? Support your answer mathematically.

4.27 Consider a 3×3 spatial mask that averages the four closest neighbors of a point (x, y) , but excludes the point itself from the average.

- Find the equivalent filter, $H(u, \nu)$, in the frequency domain.
- Show that your result is a low filter. *Hint: Express the filter as a function of $(u_0, \nu_0) = (M/2, N/2)$.*

4.33 Consider the images shown. The image on the right was obtained by: (a) multiplying the image on the left by $(-1)^{x+y}$; (b) computing the DFT; (c) taking the complex conjugate of the transform; (d) computing the inverse DFT; and (e) multiplying the real part of the result by $(-1)^{x+y}$. Explain (mathematically) why the image on the right appears as it does.

4.40 Use a Butterworth highpass filter to construct a homomorphic filter that has the same general shape as the filter in Fig. 4.61.