

Spectral Representation of Spatial Correlations in Variational
Assimilation Systems with Grid Point Models: Application to the
Belgian Assimilation System of Chemical Observations (BASCOE)

Quentin Errera

Belgium Institute for Space Aeronomy (BIRA-IASB)

quentin@oma.be

Richard Ménard

Environment Canada

SPARC DA Workshop

June 11-13, 2012

1. Formulation of the **B** matrix on a spherical harmonic basis
2. Real experiments with MIPAS (SH Winter 2003) and Eos/MLS (SH Winter 2007) observations
3. Conclusions

The classical (i.e. non-incremental) formulation of variational assimilation:

$$J(\mathbf{x}) = \underbrace{\frac{1}{2}[\mathbf{x} - \mathbf{x}^b]^T \mathbf{B}^{-1}[\mathbf{x} - \mathbf{x}^b]}_{J^b \equiv \text{background term}} + \underbrace{\frac{1}{2}[\mathbf{y} - H(\mathbf{x})]^T \mathbf{R}^{-1}[\mathbf{y} - H(\mathbf{x})]}_{J^o \equiv \text{observation term}}$$

- As the typical dimension of \mathbf{x} is around 10^6 , a full \mathbf{B} matrix is of size 10^{12} . This is far too large to deal by current computers.
- Prop: On a spherical harmonic (SH) basis, homogeneous and isotropic horizontal correlations are represented by a diagonal matrix (Boer, 1983, JAS).
- Assuming homogeneous and isotropic horizontal correlations, the spherical harmonic representation of \mathbf{B} is block diagonal

$$\mathbf{L}\chi = \mathbf{x} - \mathbf{x}^b$$

$$\mathbf{B} = \mathbf{L}\mathbf{L}^T \Leftrightarrow \mathbf{L} = \mathbf{B}^{1/2}$$

χ is defined in the spectral space and \mathbf{L} is defined by

$$\mathbf{L} = \mathbf{\Sigma}\mathbf{S}\mathbf{\Lambda}^{1/2}$$

where

- $\mathbf{\Lambda}$ is the spatial correlation matrix (block diagonal) defined in the spectral space
- \mathbf{S} is the direct spectral transform operator from the SH basis to the model grid
- $\mathbf{\Sigma}$ is the background error standard deviation matrix (diagonal)

As usual in variational assimilation, \mathbf{L}^* must also be provided

$$\mathbf{L}^* = \mathbf{\Lambda}^{1/2}\mathbf{S}^*\mathbf{\Sigma}$$

This formulation and the F90 code of the different operators are described/reviewed in

Technical Note: Spectral Representation of Spatial Correlations in Variational Assimilation with Grid Point Models and Application to the Belgian Assimilation System for Chemical Observations (BASCOE)

Quentin Errera¹ and Richard Ménard²

¹Institut d'Aéronomie Spatiale de Belgique, BIRA-IASB, Belgium

²Environment Canada, Canada

Accepted for publication in *ACPD*

Formulation: Comment about the Gaussian grid

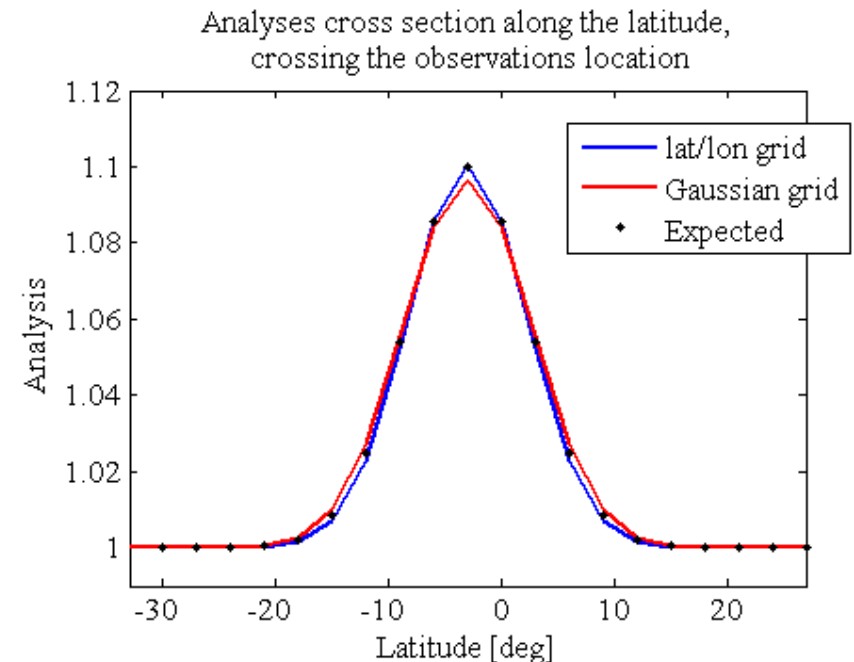
This formulation does not need the Gaussian grid

- The Gaussian grid is necessary for spectral model where some operations are done on the model grid and others in the spectral space
- \mathbf{S}^* can be built to work directly from the (lat/lon) model grid to the SH basis

→ The SH spectral method can thus be applied to any grid point model without any mapping transformation from the spectral grid to the model grid

Example: Assimilation of a single obs

- $y^o = 1.2$; $\sigma^o = 0.1$
- $x^b = 1$; $\sigma^b = 0.1$
- \mathbf{B} : Gaussian correlations; $L_h=600$ km; $L_v=3$ levels
- Expected results: Gaussian shape analysis with $L_h=600$ km and $L_v=3$ levels levels, and the value of 1.1 at the obs location



Aliasing in *spectral assimilation* less strict than in spectral modeling

- N is the truncation degree
- K is the number of latitudes
- M is the number of longitudes

- Preventing for aliasing in spectral modeling:

$$N=K-1$$

$$N=M/2 - 1$$

$$\text{i.e. } K=M/2$$

- Preventing for aliasing in *spectral assimilation*:

$$N=\max(K, M/2)$$

→ Lat and lon resolution of our model can be different

BASCOE

- 4D-Var system dedicated to stratospheric chemical observations
- CTM including 57 species, 200 chemical reactions
- PSC parameterization
- Chemistry (and PSC param.) can be turned off
- Source of Mesospheric NO_x NOT modeled
- Assimilation window is 24h

Dynamic

- ECMWF Era-Interim

Resolution

- Chemistry off: 2° long x 2° lat x 37 levels x 15 minutes
- Chemistry on: 3.75° long x 2.5° lat x 37 levels x 30 minutes

Data

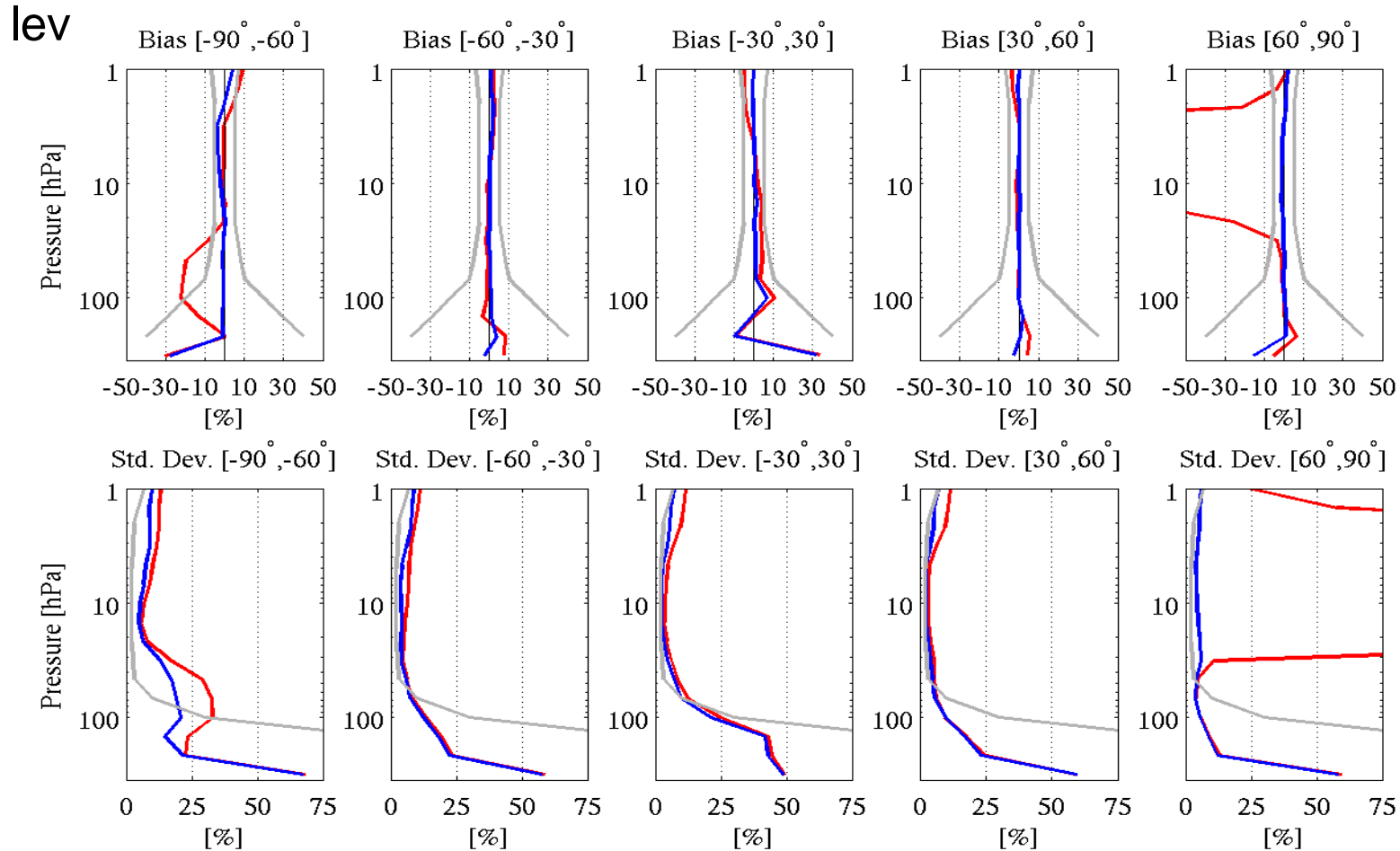
- MIPAS v4.61 ; Antarctic spring-winter 2003
- MLS v3.3 ; Antarctic spring-winter 2007

Assimilation with chemistry turned off

Ozone is assimilated as a passive tracer

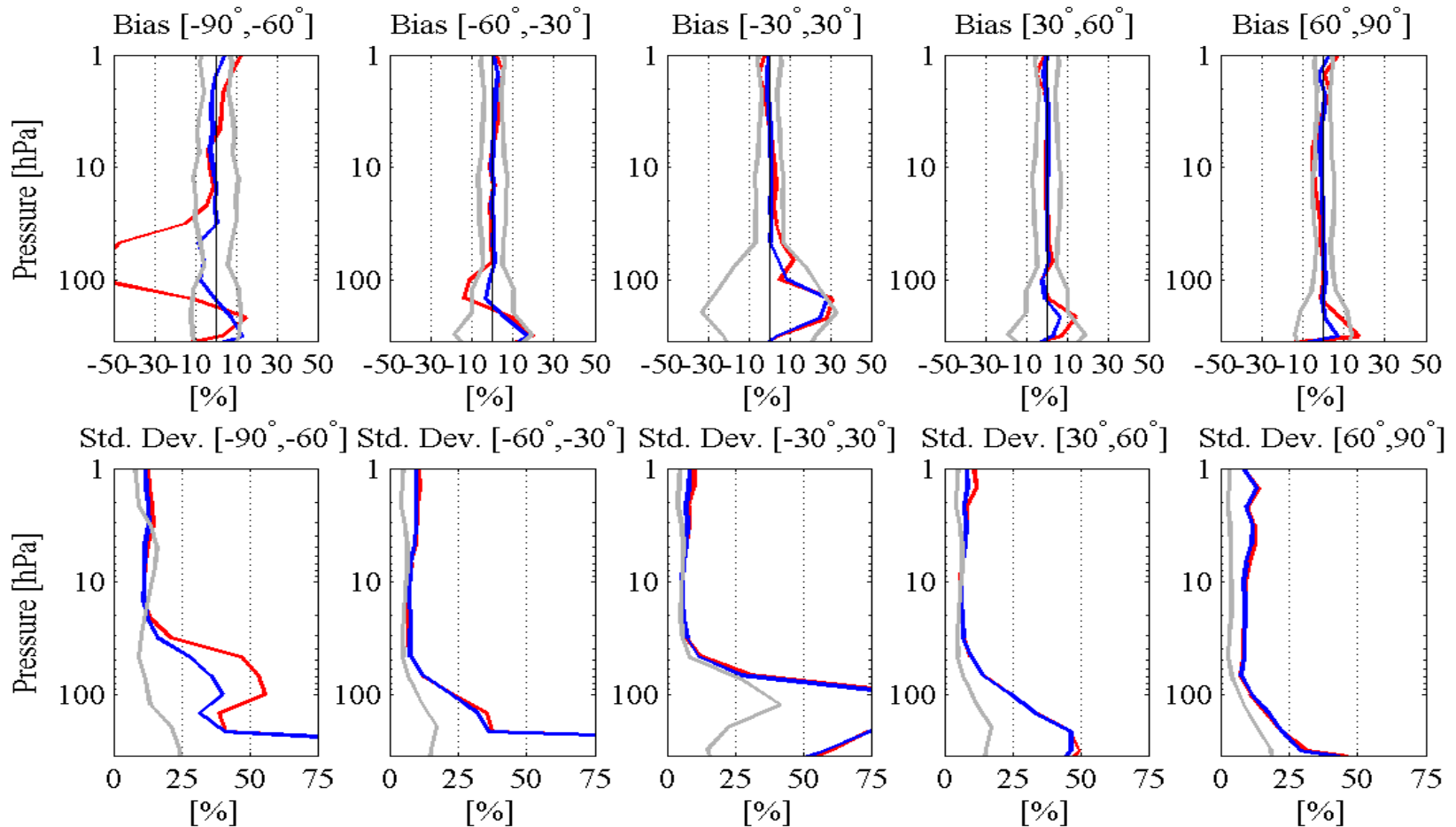
Bias and Std Dev of OmF for Sep-Oct 2007 from MLS assimilation

- **Diagonal B** : $\Sigma = 0.3 \mathbf{x}^b$
- **Correlations** : $\Sigma = 0.3 \mathbf{x}^b$, Correlation model: Gaussian, $L_h=600$ km, $L_v=1.5$ lev



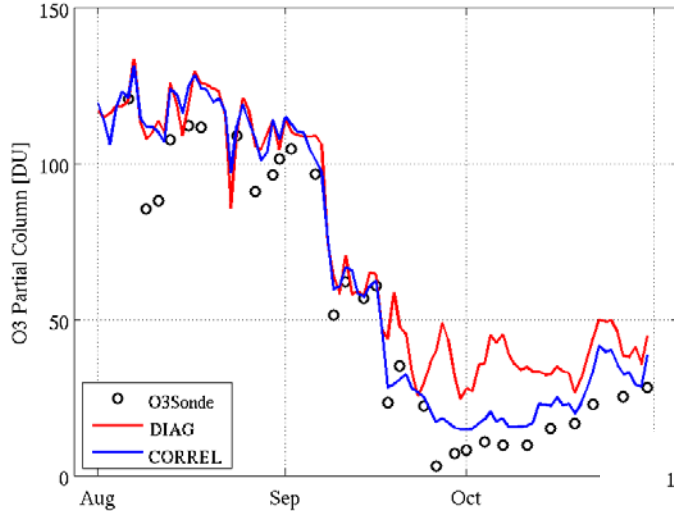
Bias and Std Dev of OmF for Sep-Oct 2003 from MIPAS assimilation

- **Diagonal B** : $\Sigma = 0.3 \mathbf{x}^b$
- **Correlations** : $\Sigma = 0.3 \mathbf{x}^b$, Correlation model: Gaussian, $L_h=600$ km, $L_v=1.5$

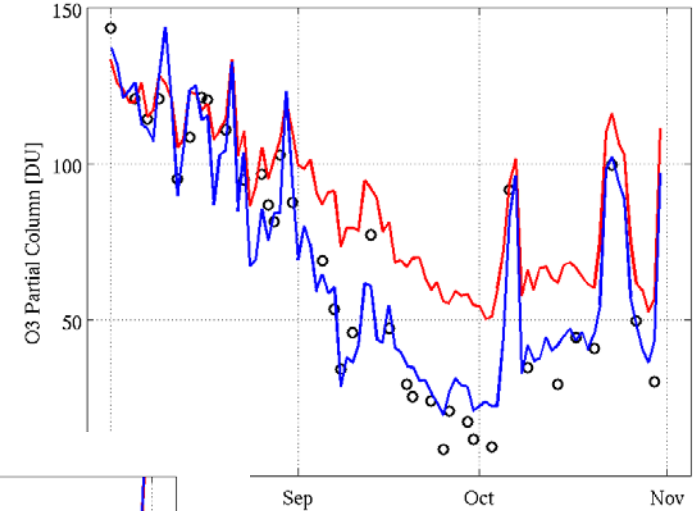


Time series of O3 partial column [10-100 hPa] above three NDACC groundbased stations in Antarctica

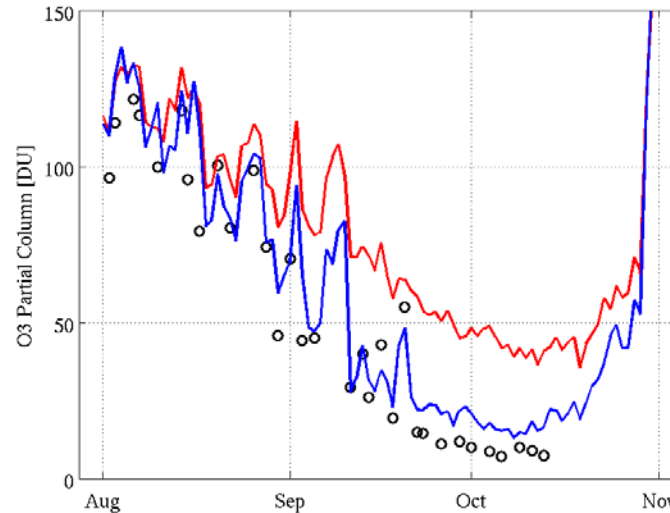
(a) Amundsen-Scott (90.0S,24.8W)



(b) Mac Murdo (77.8S,166.63E)



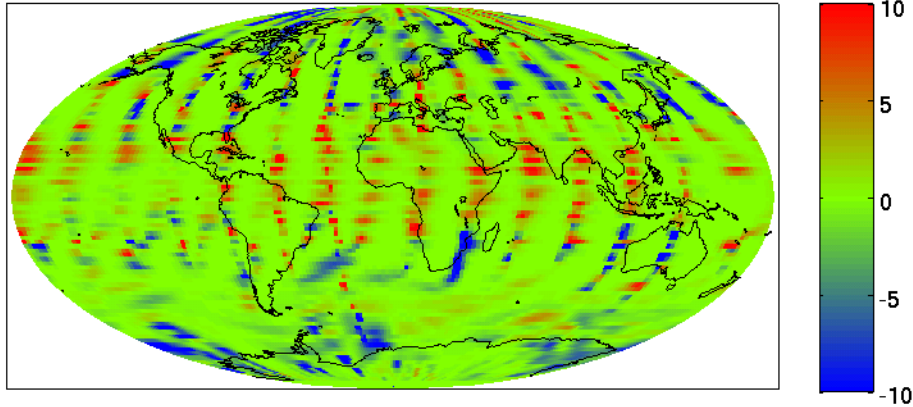
(c) Neumayer (70.7S,8.3W)



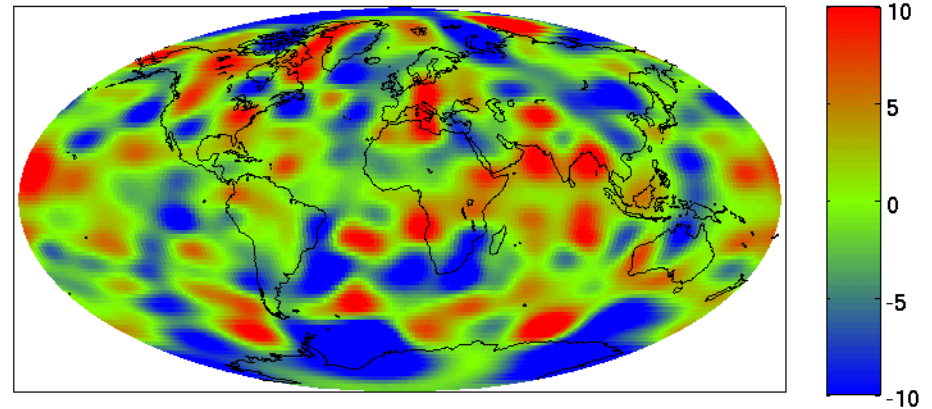
Diagonal **B**

Correlation in **B**

(a) Analysis Increments [%] for run DIAG at 44.335 hPa



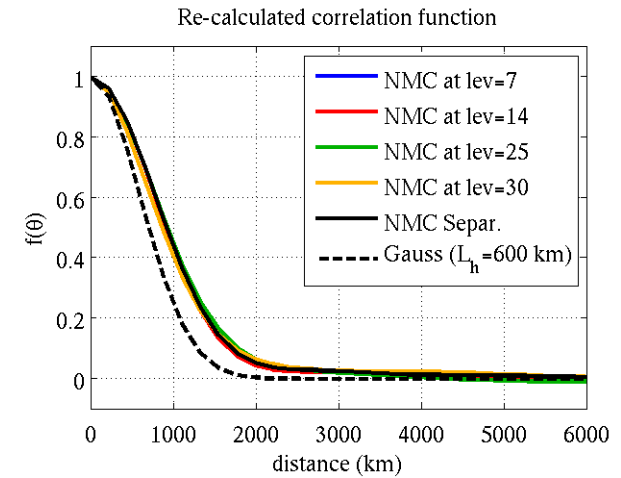
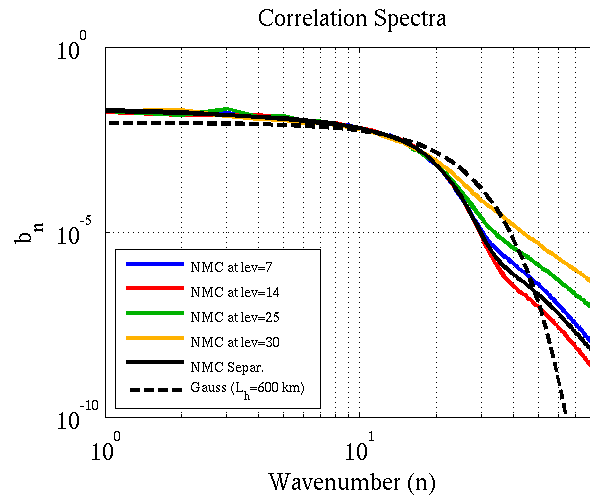
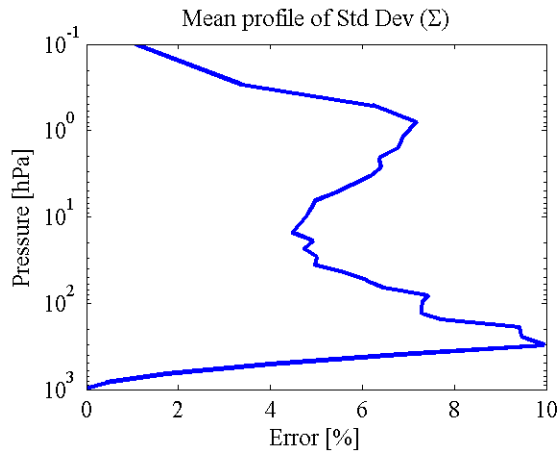
(b) Analysis Increments [%] for run CORREL at 44.335 hPa



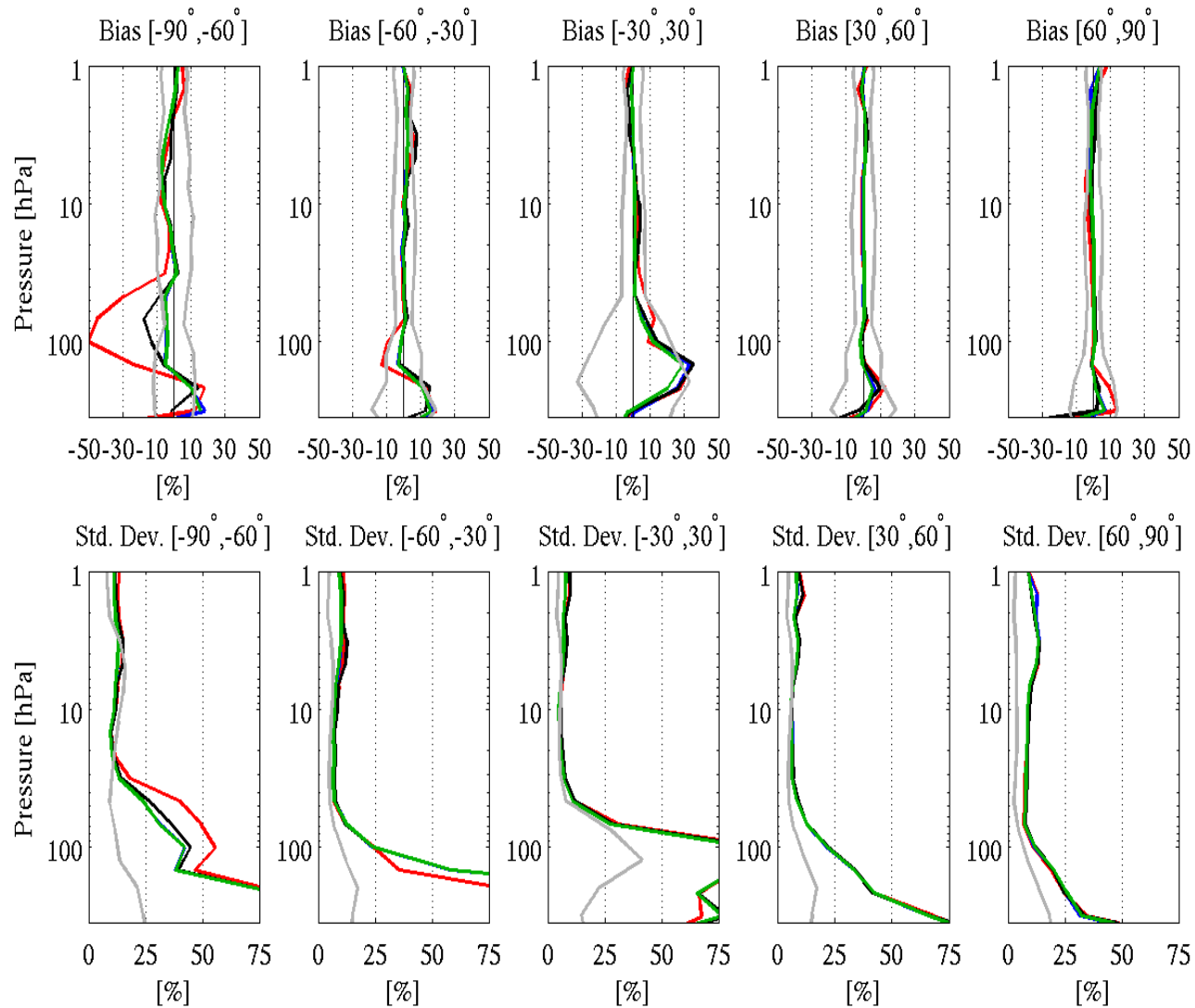
→ Correlations increase the size of the analysis increments

Calibration of \mathbf{B} (1/2): NMC method (Rabier et al., 1998, QJRMS)

1. By making an ensemble of error fields ζ , one can calibrate Σ and Λ
2. ζ are defined as $\mathbf{x}^{48h}-\mathbf{x}^{24h}$ (as in NWP) or $\mathbf{x}^{24h}-\mathbf{x}^{0h}$ (this work)
3. From that, you can estimate (here based on MIPAS O3 assimilation):



Bias and Std Dev of OmF for Sep-Oct 2003 from MIPAS assimilation



DIAG

Correlation Default

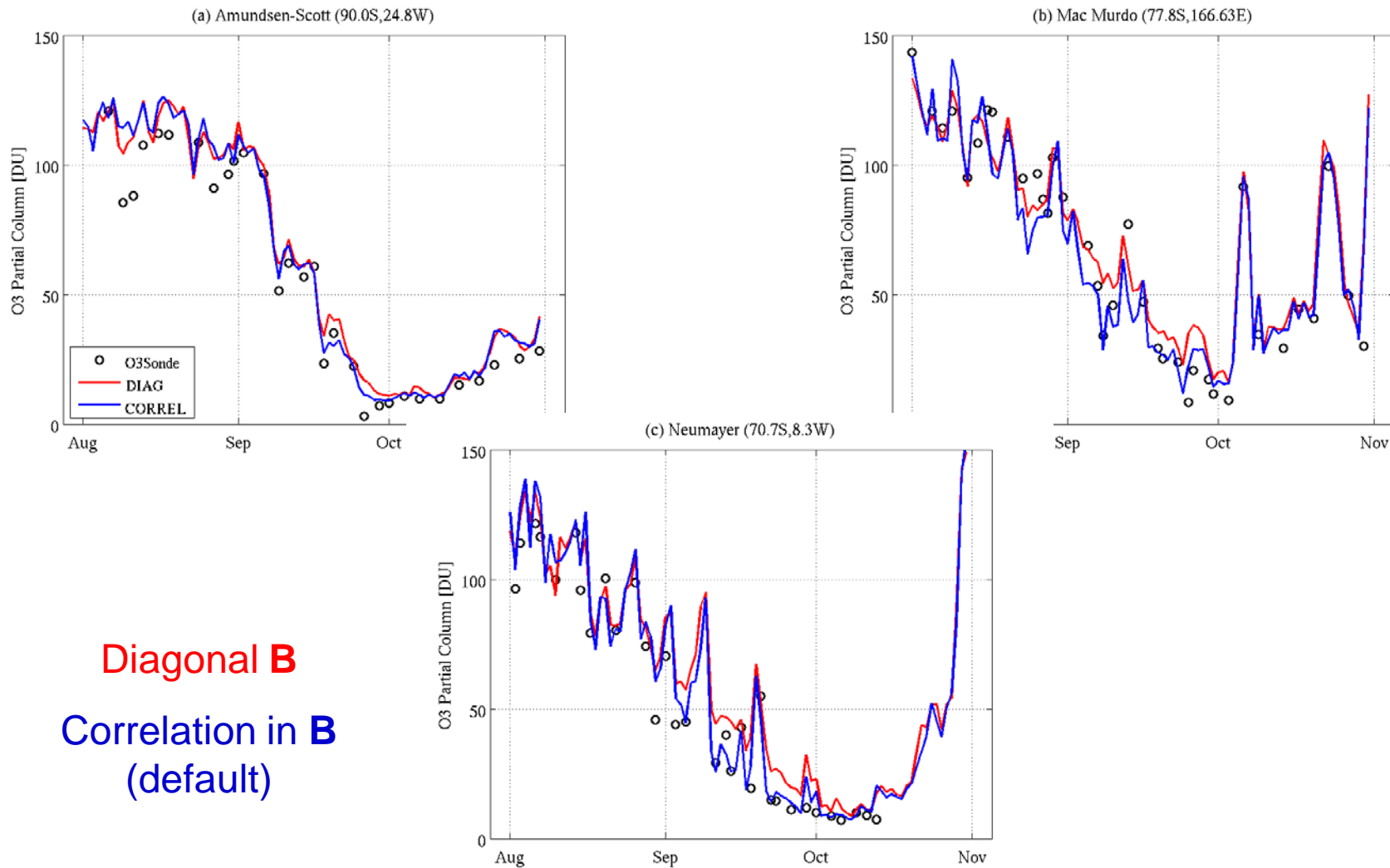
b_n : NMC; C^v : NMC; Σ : $0.3 x^b$

b_n & C^v : Default, Σ : $5*NMC$

Assimilation with chemistry turned on

All available species assimilated

Time series of O3 partial column [10-100 hPa] above three NDACC groundbased stations in Antarctica



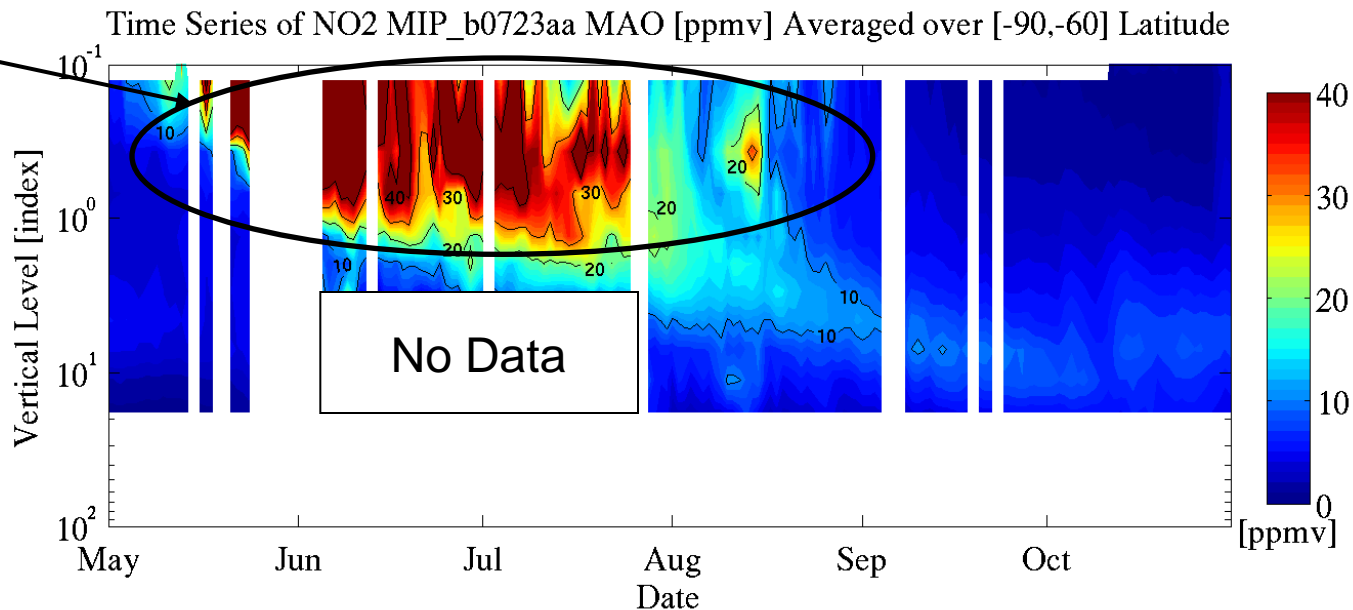
Diagonal **B**

Correlation in **B**
(default)

Assimilation of MIPAS NO2 and HNO3 (May-Nov 2003): Chemistry on

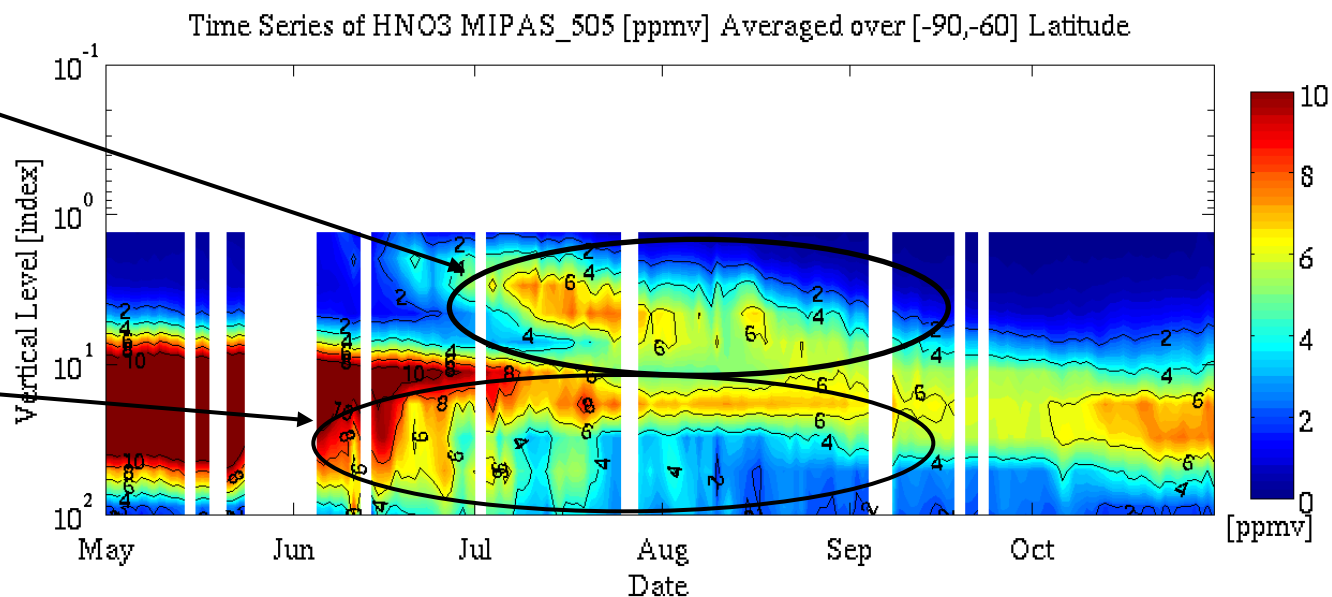
Downward transport of Mesospheric NOx produced by EPP.

Funke et al., JGR, 2005

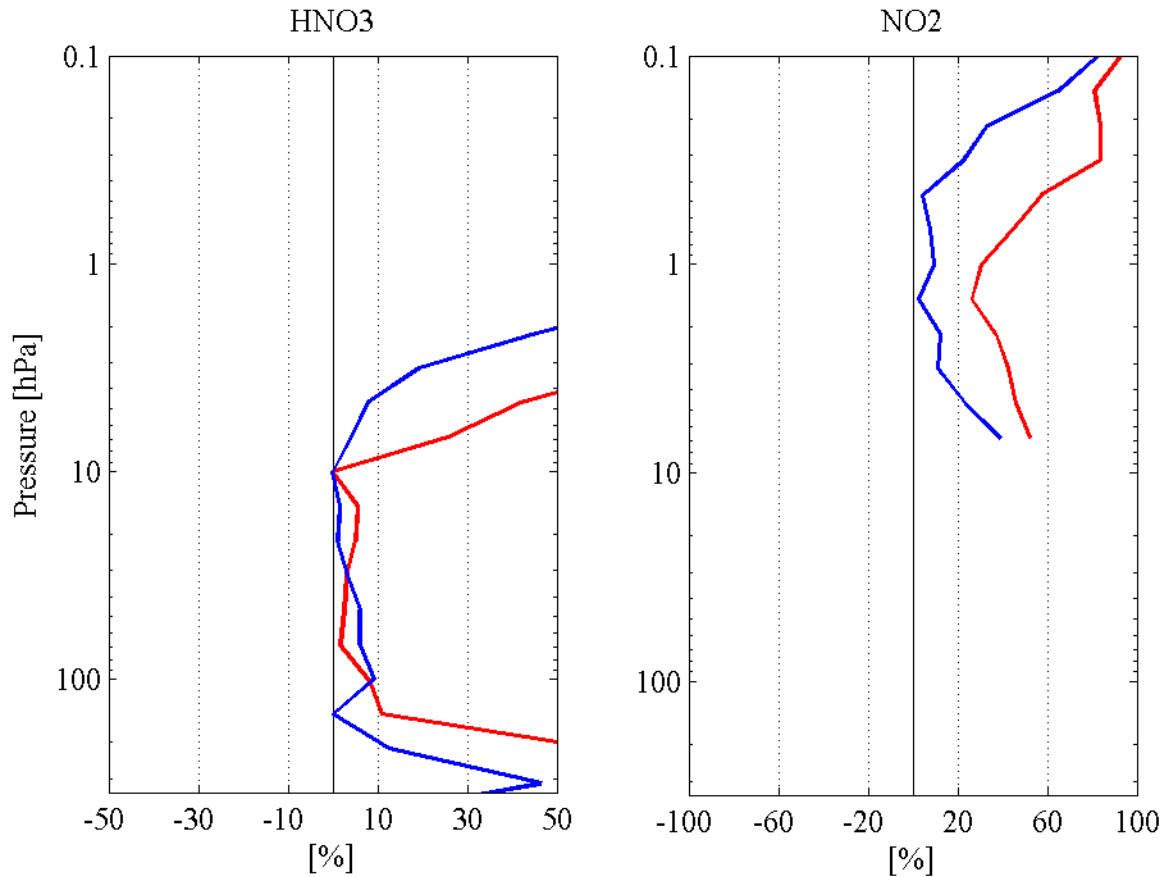


Local production of HNO3 due to NOx enhancement

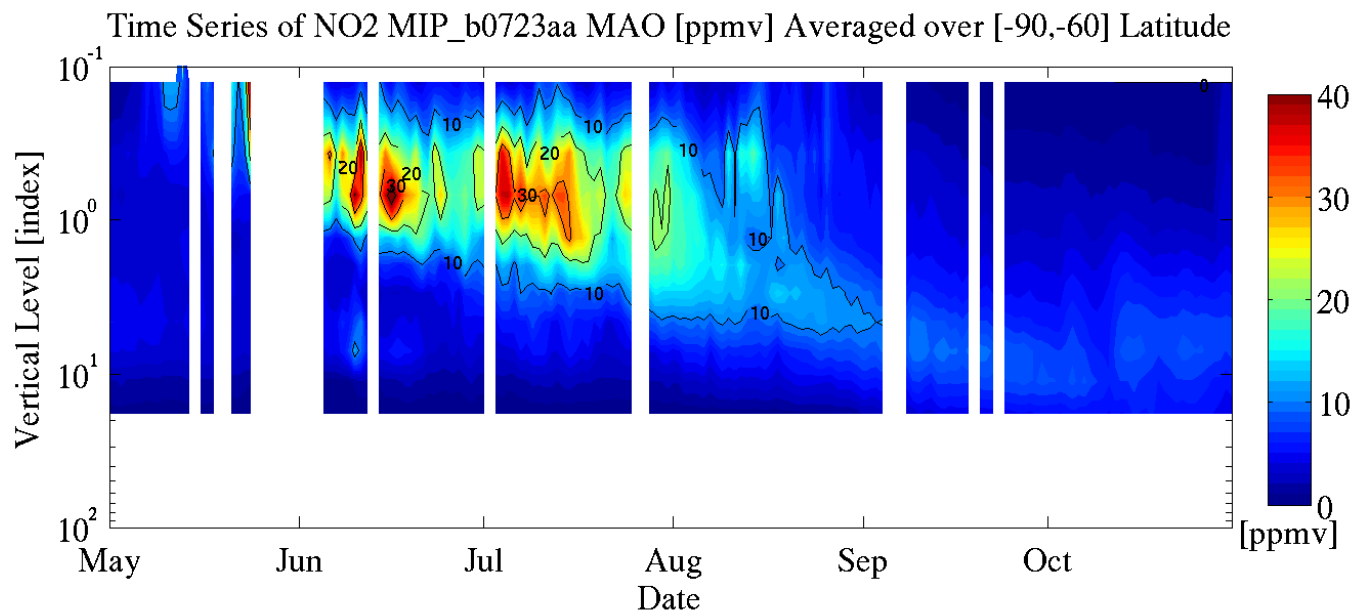
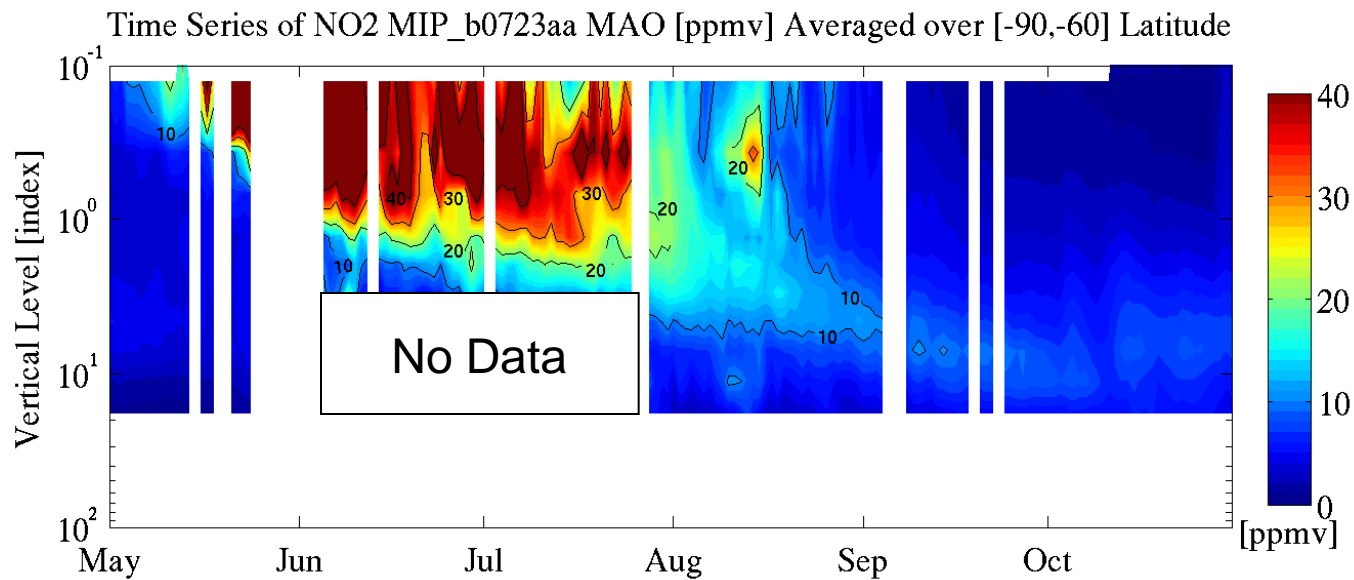
Denitrification



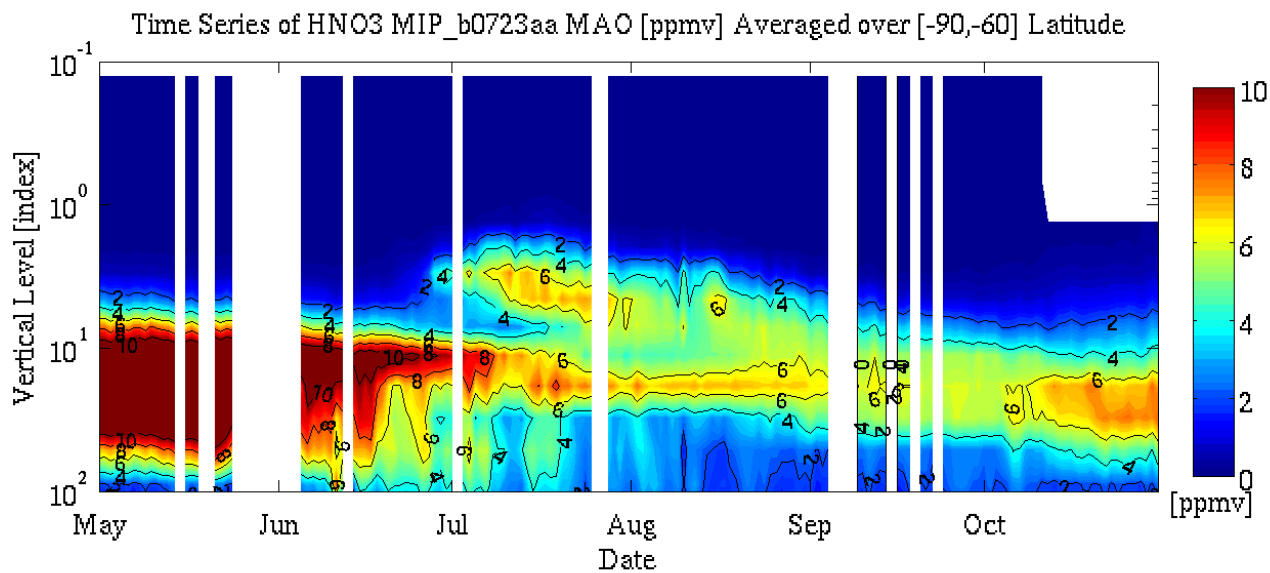
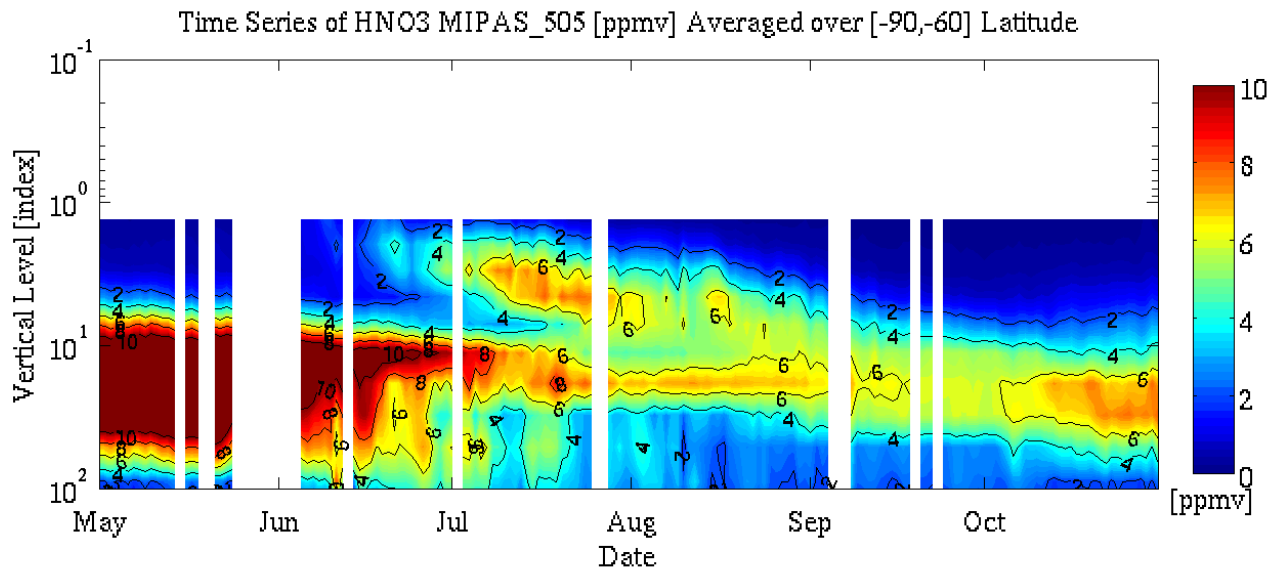
- OmF Bias for HNO₃ and NO₂ during 15Jul-15Aug 2003



Assimilation of MIPAS NO₂ and HNO₃ (May-Nov 2003): Chemistry on



Assimilation of MIPAS NO2 and HNO3 (May-Nov 2003): Chemistry on



Summary

1. New **B** matrix in BASCOE using homogeneous and isotropic horizontal correlations represented on a spherical harmonic basis
2. This implementation improves all constituent's analyses (O₃, NO₂ and HNO₃)
3. Error statistics calculated by the NMC method and implemented in BASCOE does not seem to improve the analyses

Perspective

- Reanalysis of Envisat stratospheric data using BASCOE
- List of available species:
 - MIPAS v6.00 : O₃, HNO₃, NO₂, N₂O₅, ClONO₂, N₂O, CH₄, H₂O, CFC11, CFC12
 - GOMOS : O₃, NO₂
 - SCIAMACHY : O₃, BrO

Calibration of \mathbf{B} (1/2): innovation method (Hollingsworth and Lonnberg, 1986, Tellus)

$$\mathbf{x}^b = \mathbf{x}^{\text{truth}} + \epsilon^b$$

$$\mathbf{y}^o = \mathbf{H}\mathbf{x}^{\text{truth}} + \epsilon^o$$

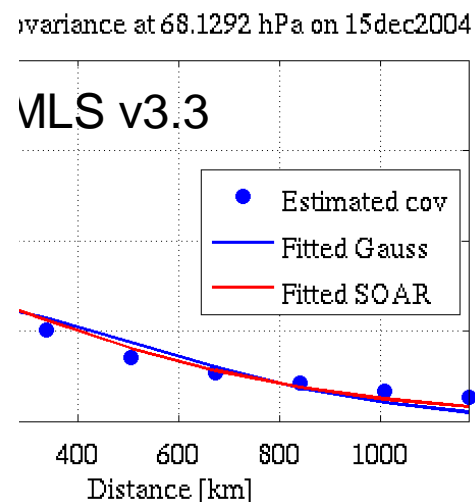
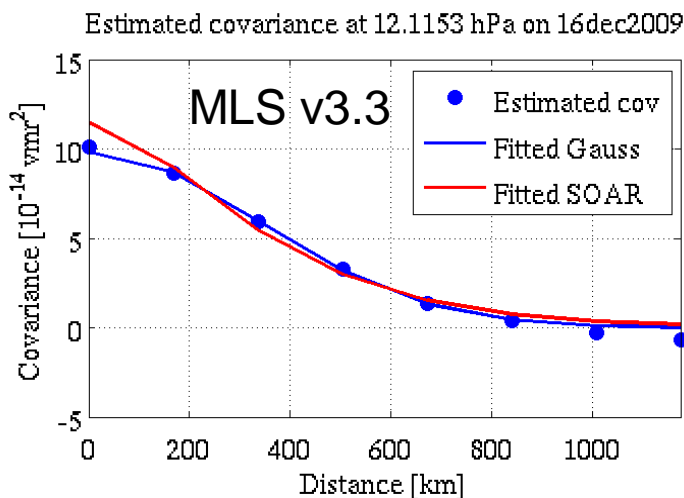
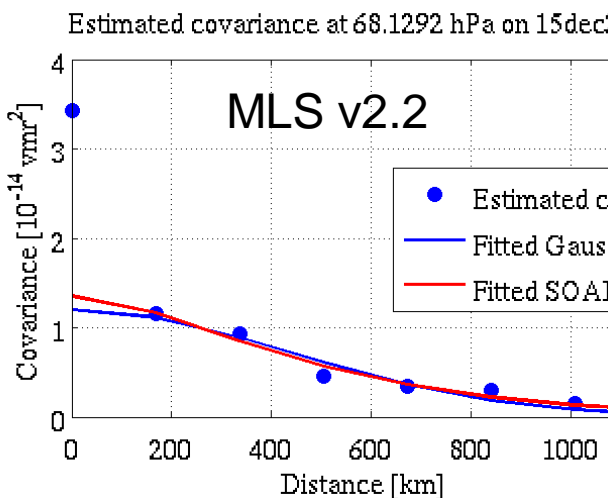
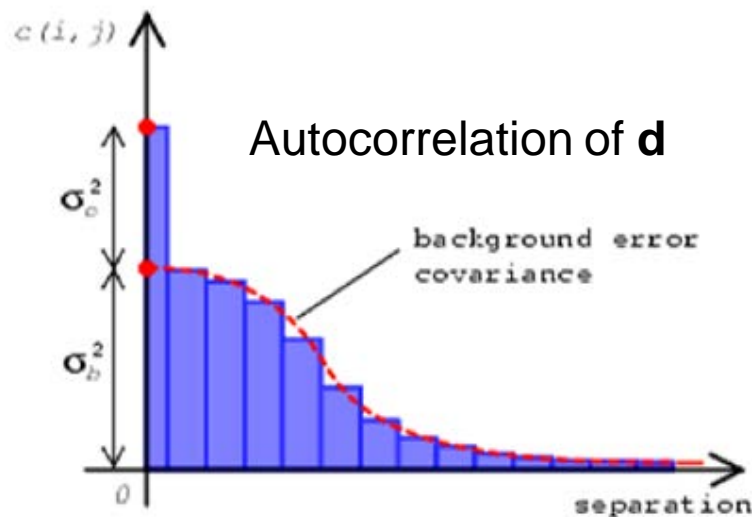
then:

$$\mathbf{d} \equiv \mathbf{y}^o - \mathbf{H}\mathbf{x}^b = \epsilon^o - \mathbf{H}\epsilon^b$$

$$\text{Thus: } \langle \mathbf{d}(i, i), \mathbf{d}(i, i) \rangle = (\sigma^b)^2 + (\sigma^o)^2$$

$$\langle \mathbf{d}(i, j), \mathbf{d}(i, j) \rangle = (\sigma^b)^2 \rho(r_{ij})$$

- Assuming that observation errors are **horizontally uncorrelated** and that background errors are spatially correlated, auto-covariance of \mathbf{d} can be used to estimate the errors



Calibration of B (2/3): innovation method

- Inefficiency of innovation method due to MLS retrieval

JPL D-33509

Earth Observing System (EOS)

Aura Microwave Limb Sounder (MLS)

Version 2.2 Level 2 data quality and description document.

In the v2.2 (*and* v3.3) Level 2 algorithms, the state vector consists of “chunks” of several profiles [...] which are then simultaneously retrieved from radiances.

