5.110. The following table summarizes some of the basic attributes of a number of BJTs of different types, operating as amplifiers under various conditions. Provide the missing entries.

<table>
<thead>
<tr>
<th>Transistor</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.98</td>
<td>1</td>
<td>0.99</td>
<td>0.984</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\infty$</td>
<td>100</td>
<td>$\infty$</td>
<td>100</td>
<td>9</td>
<td>62.8</td>
<td></td>
</tr>
<tr>
<td>$I_C$ (mA)</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
<td>0.99</td>
<td>1</td>
<td>0.25</td>
<td>4.5</td>
</tr>
<tr>
<td>$I_E$ (mA)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.02</td>
<td>1</td>
<td>0.253</td>
<td>5</td>
<td>70.2</td>
</tr>
<tr>
<td>$I_B$ (mA)</td>
<td>0</td>
<td>0.0099</td>
<td>0.0204</td>
<td>0</td>
<td>0.0025</td>
<td>0.5</td>
<td>1.10</td>
</tr>
<tr>
<td>$g_m$ (mA/V)</td>
<td>40</td>
<td>39.6</td>
<td>40</td>
<td>40</td>
<td>9.9</td>
<td>180</td>
<td>700</td>
</tr>
<tr>
<td>$r_e$ (\Omega)</td>
<td>25</td>
<td>25</td>
<td>24.5</td>
<td>25</td>
<td>100</td>
<td>5</td>
<td>1.41</td>
</tr>
<tr>
<td>$r_\pi$ (\Omega)</td>
<td>$\infty$</td>
<td>2530</td>
<td>1250</td>
<td>$\infty$</td>
<td>10.1K</td>
<td>50</td>
<td>89.7</td>
</tr>
</tbody>
</table>

(Note: Isn’t it remarkable how much two parameters can reveal?)

I will just provide the explicit equations for the first column, (a):

$$
\beta = \frac{\alpha}{1 - \alpha}
$$

$$
I_E = \frac{I_C}{\alpha}
$$

$$
I_B = \frac{I_C}{\beta}
$$

$$
g_m = \frac{I_C}{V_T}
$$

$$
r_e = \frac{\alpha}{g_m}
$$

$$
r_\pi = \frac{\beta}{g_m}
$$

5.114. A biased BJT operates as a grounded-emitter amplifier between a signal source, with a source resistance of 10 k\Omega, connected to the base and a 10 k\Omega
load connected as a collector resistance $R_C$. In the corresponding model, $g_m$ is $40 \text{ mA/V}$ and $r_\pi$ is $2.5 \text{ k}\Omega$. Draw the complete amplifier model using the hybrid-$\pi$ BJT equivalent circuit. Calculate the overall voltage gain $v_c/v_s$. What is the value of BJT $\beta$ implied by the values of the model parameters? To what value must $\beta$ be increased to double the overall voltage gain?

$$v_o = -g_m R_C v_i = -g_m R_C \left( \frac{r_\pi}{R_s + r_\pi} \right) v_s = -0.04 \times 10 \times 10^3 \times \frac{2.5}{10 + 2.5} = -80$$

The value of $\beta$ can be found from

$$r_\pi = \frac{\beta}{g_m}$$

or

$$\beta = r_\pi g_m = 2.5 \times 10^3 \times 0.04 = 100$$

To double the gain we would want to double the factor

$$\left( \frac{r_\pi}{R_s + r_\pi} \right)$$

It is currently equal to

$$\frac{2.5}{10 + 2.5} = 0.2$$

To double it we would need to change $r_\pi$:

$$\frac{r_\pi}{R_s + r_\pi} = 0.4$$

$$r_\pi = \frac{0.4}{0.6} R_s = 0.67 R_s \approx 6.7 \text{ k}\Omega$$

The factor increase in $\beta$ is the same as the factor increase in $r_\pi$:

$$\beta_{\text{new}} = \beta_{\text{old}} \frac{r_\pi_{\text{new}}}{r_\pi_{\text{old}}} = 100 \times \frac{6.7}{2.5} = 268$$

5.115. For the circuit shown in Fig. P5.115, draw a complete small-signal equivalent circuit utilizing an appropriate $T$ model for the BJT (use $\alpha = 0.99$). Your
circuit should show the values of all components, including the model parameters. What is the input resistance $R_{\text{in}}$? Calculate the overall gain $v_o/v_{\text{sig}}$.

![Circuit Diagram](image)

**FIGURE P5.115**

Here is the small-signal equivalent

where

The overall voltage can be found from

$$v_o = - (R_C || R_L) i_c$$

$$i_c = \alpha i_e$$

$$i_e = - \frac{v_{\text{sig}}}{R_{\text{sig}} + r_e}$$

so
\[
\frac{v_o}{v_{\text{sig}}} = (R_C || R_L) \frac{\alpha}{R_{\text{sig}} + r_e} \frac{1}{\frac{1}{10} \cdot 50 + 50} = 49.5
\]

5.128. A common-emitter amplifier of the type shown in Fig 5.60(a) is biased to operate at \( I_C = 0.2 \text{ mA} \) and has a collector resistance \( R_C = 24 \text{ k}\Omega \). The transistor has \( \beta = 100 \) and a large \( V_A \). The signal source is directly coupled to the base and \( C_c1 \) and \( R_B \) are eliminated. Find \( R_{\text{in}} \), the voltage gain \( A_{vo} \), and \( R_o \). Use these results to determine the overall voltage gain when a 10 k\( \Omega \) resistor is connected to the collector and the source resistance \( R_{\text{sig}} = 10 \text{ k}\Omega \).

![Common-emitter amplifier diagram](image)

This is a common-emitter amplifier with \( R_B = \infty \), so the input resistance is

\[
R_{\text{in}} = r_\pi = \frac{\beta V_T}{I_C} = \frac{100 \times 25}{0.2} = 12.5 \text{ k}\Omega
\]

\[
A_{vo} = -g_mR_C = -\frac{I_C}{V_T}R_C = -\frac{0.2}{25} \times 10^3 = -80
\]
\[ R_O = R_C = 10 \, \text{k}\Omega \]

The overall voltage gain, \( v_o/v_{\text{sig}} \) is then

\[
G_v = \frac{v_o}{v_{\text{sig}}} = \frac{r_\pi}{R_{\text{sig}} + r_\pi} \frac{A_{\text{vo}} R_L}{R_L + R_O} = \frac{12.5}{10 + 12.5} \frac{-80}{10 + 10} = -22.2
\]

5.130. For the common-emitter amplifier shown in Figure P5.130, let \( V_{\text{CC}} = 9 \, \text{V} \), \( R_1 = 27 \, \text{k}\Omega \), \( R_2 = 15 \, \text{k}\Omega \), \( R_E = 1.2 \, \text{k}\Omega \), and \( R_C = 2.2 \, \text{k}\Omega \). The transistor has \( \beta = 100 \), and \( V_A = 100 \, \text{V} \). Calculate the dc bias current \( I_E \). If the amplifier operates between a source for which \( R_{\text{sig}} = 10 \, \text{k}\Omega \) and a load of \( 2 \, \text{k}\Omega \), replace the transistor with its hybrid-\( \pi \) model, and find the values of \( R_{\text{in}} \), the voltage gain \( v_o/v_{\text{sig}} \), and the current gain \( i_o/i_i \).

We have

\[ V_B = (i_1 - i_B) R_2 \quad V_B = V_{\text{BE}} + (\beta + 1) R_E i_B \quad V_B = V_{\text{CC}} - i_1 R_1 \]

and we can eliminate \( i_D \) and \( i_B \) from the first equation.

\[
V_B = \left( \frac{V_{\text{CC}} - V_B}{R_1} - \frac{V_B - V_{\text{BE}}}{(\beta + 1) R_E} \right) R_2
\]

\[
V_B \left[ 1 + \frac{R_2}{R_1} + \frac{R_2}{(\beta + 1) R_E} \right] = V_{\text{CC}} \frac{R_2}{R_1} + V_{\text{BE}} \frac{R_2}{(\beta + 1) R_E}
\]

\[
V_B = \frac{V_{\text{CC}} R_2}{1 + \frac{R_2}{R_1} + \frac{R_2}{(\beta + 1) R_E}} = \frac{9 \times \frac{15}{27} + 0.7 \times \frac{15}{101\times1.2}}{1 + \frac{15}{27} + \frac{15}{101\times1.2}} = 3.03 \, \text{V}
\]
Note that for $(\beta + 1) R_E \gg R_2$ and $V_{CC} \gg V_{BE}$ the expression for $V_B$ reduces to the voltage divider expression,

$$V_B = V_{CC} \frac{R_2}{1 + \frac{R_2}{R_1}} = V_{CC} \frac{R_2}{R_1 + R_2} = 9 \times \frac{15}{15 + 27} = 3.21 \text{ V}$$

I will proceed with the result from the fully correct expression. Next, the emitter current is

$$I_E = \frac{V_E}{R_E} = \frac{V_B - V_{BE}}{R_E} = \frac{3.03 - 0.7}{1.2} = 1.94 \text{ mA}$$

The small-signal model looks like this

The input resistance is

$$R_{in} = R_1 || R_2 || r_\pi$$

where

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{\beta V_T}{\alpha I_E} = \frac{\beta + 1) V_T}{I_E} = \frac{101 \times 25}{1.94} = 1302 \Omega$$

such that

$$R_{in} = R_1 || R_2 || r_\pi = 27 || 15 || 1.302 = 1.15 \text{ k\Omega}$$

The voltage gain is

$$G_v = -\frac{R_{in}}{R_{in} + R_s} A_v = -\frac{R_{in}}{R_{in} + R_s} g_m (R_C || r_o || R_L) = -\frac{R_{in}}{R_{in} + R_s} \frac{\beta I_E}{(\beta + 1) V_T} (R_C || r_o || R_L)$$

where

$$r_o = \frac{V_A}{I_C} = \frac{V_A}{\alpha I_E} = \frac{\beta + 1) V_A}{\beta I_E} = \frac{101 \times 100}{100 \times 1.94 \times 10^{-3}} = 52.1 \text{ k\Omega}$$

and then
\[ G_v = -\frac{R_{in}}{R_{in} + R_s} \frac{\beta I_E}{(\beta + 1)V_T} (R_C||r_o||R_L) \]
\[ = -\frac{1.14 \times 100 \times 1.94}{1.14 + 10 \times 101 \times 25} (2.2||52.1||2) \]
\[ = -8.07 \]

5.131. Using the topology of Fig. P5.130, design an amplifier to operate between a 10 kΩ source and a 2 kΩ load with a gain \( v_o/v_{sig} \) of \(-8\). The power supply available is 9 V. Use an emitter current of approximately 2 mA and a current of about one-tenth of that in the voltage divider that feeds the base, with the dc voltage at the base about one third of the supply. The transistor available has \( \beta = 100 \) and \( V_A = 100 \) V. Use standard 5% resistors (See Appendix G)

This is the same circuit as before, except choose the nearest 5% resistors. In that case choose \( R_1 = 27 \) kΩ, \( R_2 = 15 \) kΩ, \( R_E = 1.2 \) kΩ, \( R_C = 2.2 \) kΩ. Well, it turns out the problem is identical to P5.130. That was easy.