

Lecture

Aided INS

EE 565: Position, Navigation, and Timing

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Aly El-Osery and Kevin Wedeward, Electrical Engineering Dept., New Mexico Tech
In collaboration with
Stephen Bruder, Electrical & Computer Engineering, Embry-Riddle Aeronautical University

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1 Overview

1.1 ECEF as and Example

Notation Used

- Truth value

$$\vec{x}$$

- Measured value

$$\tilde{\vec{x}}$$

- Estimated or computed value

$$\hat{\vec{x}}$$

- Error

$$\delta\vec{x} = \vec{x} - \hat{\vec{x}}$$

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1.2 Inertial Measurements

Actual Measurements

Initially the accelerometer and gyroscope measurements, $\tilde{\vec{f}}_{ib}^b$ and $\tilde{\vec{\omega}}_{ib}^b$, respectively, will be modeled as

$$\tilde{\vec{f}}_{ib}^b = \vec{f}_{ib}^b + \Delta\vec{f}_{ib}^b = \hat{\vec{f}}_{ib}^b + \Delta\hat{\vec{f}}_{ib}^b \quad (1)$$

$$\tilde{\vec{\omega}}_{ib}^b = \vec{\omega}_{ib}^b + \Delta\vec{\omega}_{ib}^b = \hat{\vec{\omega}}_{ib}^b + \Delta\hat{\vec{\omega}}_{ib}^b \quad (2)$$

where \vec{f}_{ib}^b and $\vec{\omega}_{ib}^b$ are the specific force and angular rates, respectively; and $\Delta\vec{f}_{ib}^b$ and $\Delta\vec{\omega}_{ib}^b$ represents the errors. In later lectures we will discuss more detailed description of these errors.

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Error Modeling Example

Accelerometers

$$\tilde{\vec{f}}_{ib}^b = \vec{b}_a + (\mathcal{I} + M_a)\vec{f}_{ib}^b + \vec{n}_a + \vec{w}_a \quad (3)$$

Gyroscopes

$$\tilde{\omega}_{ib}^b = \vec{b}_g + (\mathcal{I} + M_g)\tilde{\omega}_{ib}^b + G_g \vec{f}_{ib}^b + \vec{w}_g \quad (4)$$

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Pos, Vel, Force and Angular Rate Errors

- Position error

$$\delta \vec{r}_{\beta b}^\gamma = \vec{r}_{\beta b}^\gamma - \hat{\vec{r}}_{\beta b}^\gamma \quad (5)$$

- Velocity error

$$\delta \vec{v}_{\beta b}^\gamma = \vec{v}_{\beta b}^\gamma - \hat{\vec{v}}_{\beta b}^\gamma \quad (6)$$

- Specific force errors

$$\delta \vec{f}_{ib}^b = \vec{f}_{ib}^b - \hat{\vec{f}}_{ib}^b \quad (7)$$

$$\Delta_e \vec{f}_{ib}^b = \Delta \vec{f}_{ib}^b - \Delta \hat{\vec{f}}_{ib}^b = -\delta \vec{f}_{ib}^b \quad (8)$$

- Angular rate errors

$$\delta \vec{\omega}_{ib}^b = \vec{\omega}_{ib}^b - \hat{\vec{\omega}}_{ib}^b \quad (9)$$

$$\Delta_e \vec{\omega}_{ib}^b = \Delta \vec{\omega}_{ib}^b - \Delta \hat{\vec{\omega}}_{ib}^b = -\delta \vec{\omega}_{ib}^b \quad (10)$$

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ECEF Error Mechanization

Recall

$$\begin{pmatrix} \delta \dot{\psi}_{eb}^e \\ \delta \dot{\vec{v}}_{eb}^e \\ \delta \dot{\vec{r}}_{eb}^e \end{pmatrix} = \begin{bmatrix} -\Omega_{ie}^e & 0_{3 \times 3} & 0_{3 \times 3} \\ -[\hat{C}_b^e \hat{\vec{f}}_{ib}^b \times] & -2\Omega_{ie}^e & \frac{2g_0(\hat{L}_b)}{r_{eS}^e(\hat{L}_b)} \frac{\hat{\vec{r}}_{eb}^e}{|\hat{\vec{r}}_{eb}^e|^2} (\hat{\vec{r}}_{eb}^e)^T \\ 0_{3 \times 3} & \mathcal{I}_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \begin{pmatrix} \delta \psi_{eb}^e \\ \delta \vec{v}_{eb}^e \\ \delta \vec{r}_{eb}^e \end{pmatrix} + \begin{bmatrix} 0 & -\hat{C}_b^e \\ -\hat{C}_b^e & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta_e \vec{f}_{ib}^b \\ \Delta_e \vec{\omega}_{ib}^b \end{pmatrix} \quad (11)$$

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Errors After Calibration

In reality there will be error terms in the sensor that can not be calibrated. These terms may be estimated. The error in the estimation of these terms may be expressed as

$$\Delta_e \vec{f}_{ib}^b = \Delta \vec{f}_{ib}^b - \Delta \hat{\vec{f}}_{ib}^b = F_{va} \delta \vec{x}_a + \vec{\zeta}_a \quad (12)$$

$$\Delta_e \vec{\omega}_{ib}^b = \Delta \vec{\omega}_{ib}^b - \Delta \hat{\vec{\omega}}_{ib}^b = F_{\psi g} \delta \vec{x}_g + \vec{\zeta}_g \quad (13)$$

These terms represent the difference between what we estimate the errors in the sensors to be (either through calibration or online estimation) and the actual errors in the sensor.

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Error Terms

The matrices F_{va} and $F_{\psi g}$, depend on the needed level of complexity in modeling the errors. For example if we only model biases, e.g., $\delta \vec{x}_a = \delta \vec{b}_a$, then $F_{va} = \mathcal{I}$.

If more error terms are modeled, then most likely, we will end up with non-linear equations, and therefore linearization is necessary.

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Error Modeling

$$\delta \dot{\vec{x}}_a = F_{aa} \delta \vec{x}_a + \vec{w}_a \quad (14)$$

$$\delta \dot{\vec{x}}_g = F_{gg} \delta \vec{x}_g + \vec{w}_g \quad (15)$$

The matrices F_{aa} and F_{gg} are specific to accelerometers and the gyroscopes and there specific configuration within the IMU.

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State Augmentation

After state augmentation

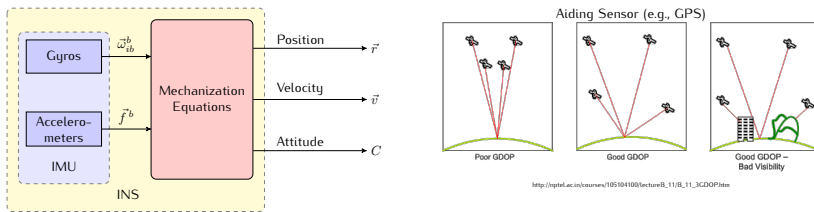
$$\begin{pmatrix} \delta \dot{\psi}_{eb}^e \\ \delta \dot{v}_{eb}^e \\ \delta \dot{r}_{eb}^e \\ \delta \dot{x}_a \\ \delta \dot{x}_g \end{pmatrix} = \begin{bmatrix} -\Omega_{ie}^e & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & -\hat{C}_b^e F_{\psi g} \\ -[\hat{C}_b^e \hat{f}_{ib}^b \times] & -2\Omega_{ie}^e & \frac{2g_0(\hat{L}_b)}{r_{eS}^e(\hat{L}_b)} \frac{\hat{r}_{eb}^e}{|\hat{r}_{eb}^e|^2} (\hat{r}_{eb}^e)^T & -\hat{C}_b^e F_{va} & 0_{3 \times 3} \\ 0_{3 \times 3} & \mathcal{I}_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & F_{aa} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & F_{gg} \end{bmatrix} \begin{pmatrix} \delta \psi_{eb}^e \\ \delta v_{eb}^e \\ \delta r_{eb}^e \\ \delta x_a \\ \delta x_g \end{pmatrix} + \begin{bmatrix} -\hat{C}_b^e & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & -\hat{C}_b^e & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & \mathcal{I}_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & \mathcal{I}_{3 \times 3} \end{bmatrix} \begin{pmatrix} \vec{\zeta}_g \\ \vec{\zeta}_a \\ 0_{3 \times 1} \\ \vec{w}_a \\ \vec{w}_g \end{pmatrix} \\ = F(t)\vec{x} + G\vec{w}$$

(16)

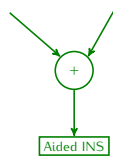
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1.3 Background

Need for Integration



Advantages	Disadvantages
<ul style="list-style-type: none"> Immune to RF Jamming High data rate High accuracy in short term 	<ul style="list-style-type: none"> Drifts Errors are time dependent Need Initialization

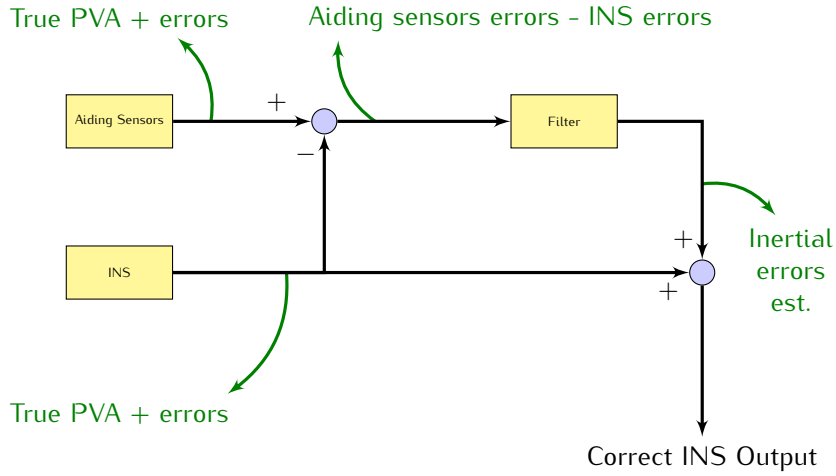


Advantages	Disadvantages
<ul style="list-style-type: none"> Errors time-indep. No initialization 	<ul style="list-style-type: none"> Sensitive to RF Interference No attitude information

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2 Integration Architectures

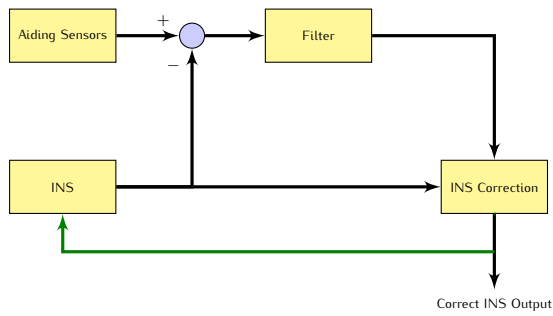
Open-Loop Integration



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Closed-Loop Integration

If error estimates are fed back to correct the INS mechanization, a reset of the state estimates becomes necessary.



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3 Integration Filter

Kalman Filter

$$\hat{\vec{x}}_{k|k-1} = \Phi_{k-1} \hat{\vec{x}}_{k-1|k-1} \quad (17)$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \Phi_{k-1} \mathbf{P}_{k-1|k-1} \Phi_{k-1}^T \quad (18)$$

$$\hat{\vec{x}}_{k|k} = \hat{\vec{x}}_{k|k-1} + \mathbf{K}_k (\vec{z}_k - \mathbf{H}_k \hat{\vec{x}}_{k|k-1}) \quad (19)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \quad (20)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (21)$$

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Closed-Loop Kalman Filter

Since the errors are being fed back to correct the INS, the state estimate must be reset after each INS correction.

$$\hat{x}_{k|k-1} = 0 \quad (22)$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \Phi_{k-1} \mathbf{P}_{k-1|k-1} \Phi_{k-1}^T \quad (23)$$

$$\hat{x}_{k|k} = \mathbf{K}_k \bar{z}_k \quad (24)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \quad (25)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (26)$$

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Discretization

$$\Phi_{k-1} \approx \mathbf{I} + \mathbf{F} \Delta t \quad (27)$$

$$\mathbf{Q} = \begin{pmatrix} n_{rg}^2 \mathbf{I}_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & n_{ag}^2 \mathbf{I}_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & n_{bad}^2 \mathbf{I}_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & n_{bgd}^2 \mathbf{I}_{3 \times 3} \end{pmatrix} \quad (28)$$

where Δt is the sample time, n_{rg}^2 , n_{ag}^2 , n_{bad}^2 , n_{bgd}^2 are the PSD of the gyro and accel random noise, and accel and gyro bias variation, respectively.

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Discrete Covariance Matrix \mathbf{Q}_k

Assuming white noise, small time step, \mathbf{G} is constant over the integration period, and the trapezoidal integration

$$\mathbf{Q}_{k-1} \approx \frac{1}{2} [\Phi_{k-1} \mathbf{G}_{k-1} \mathbf{Q}(t_{k-1}) \mathbf{G}_{k-1}^T \Phi_{k-1}^T + \mathbf{G}_{k-1} \mathbf{Q}(t_{k-1}) \mathbf{G}_{k-1}^T] \Delta t \quad (29)$$

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