EE 565: Position, Navigation and Timing Error Modeling

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Given a state space system

$$\dot{\vec{x}}(t) = F(t)\vec{x}(t) + G(t)\vec{w}(t) \tag{1}$$

and the measurement equation

$$\vec{y}(t) = H(t)\vec{x}(t) + \vec{v}(t) \tag{2}$$

If \vec{w} and \vec{v} are Gaussian random processes, then the system above is known as Gauss-Markov process. This process is used to describe a variety of error models of interest. Particularly,

- random constant.
- random walk, and
- first order Markov.

Random Constants



Assuming that the error is due to an unknown constant (i.e. random constant), the following model is used

$$\dot{\vec{x}}(t) = 0 \tag{3}$$

This model is useful in describing effects such as bias stability, scale factor errors, and misalignments. The value can vary from turn-on to turn-on.



An INS system requires the integration of accel and gyro measurements. This leads to the Gauss-Markov process

$$x(t) = \int_0^t w(\nu) d\nu \Rightarrow \dot{x} = w(t)$$

with

$$\mathbb{E}\{w(t)\}=0\tag{4}$$

$$\mathbb{E}\{w(t)w(t+\tau)\} = Q\delta(\tau) \tag{5}$$

$$\mathbb{E}\{x(t)\}=0$$

$$\mathbb{E}\{x^2(t)\} = Qt$$

called random walk.

Random Walk Cont.



- In the case of the gyroscope, w would be in the rate domain and thus has units of deg/s and the units of x(t) is degrees.
- $\mathbb{E}\{x^2(t)\} = Qt$ has units of deg^2 .
- Therefore, Q has the units of $\frac{deg^2}{s} = \frac{(deg/s)^2}{Hz}$ which is the PSD level of the gyro white noise.
- This matches the units in Eq. 5 (Note that $\delta(\tau)$ has units of 1/time).
- In simulation the power of the white noise w that is experienced by the system is F_sQ , where F_s is the sampling rate.
- Consquently, $\sigma_w = \sqrt{F_s Q}$ and has the units of deg/s
- Recall that the gyro angle random walk (ARW)

$$ARW(^{\circ}/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((^{\circ}/h)^2/Hz)}$$
 (6)



Frequently used to model bias instability (BI)

State Equation

$$\dot{b}(t) = -\frac{1}{T_c}b(t) + w(t) \tag{7}$$

Autocorrelation

$$\mathbb{E}\{b(t)b(t+\tau)\} = \sigma_{BI}^2 e^{-|\tau|/T_c}$$
(8)

where

$$\mathbb{E}\{w(t)w(t+\tau)\}=Q\delta(t-\tau)$$

$$Q = \frac{2\sigma_{BI}^2}{T}$$

and T_c is the correlation time.



State Equation

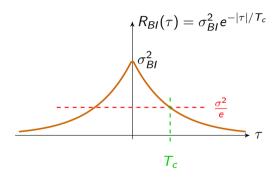
$$b_k = e^{-\frac{1}{T_c}\tau_s}b_{k-1} + w_{k-1} \tag{11}$$

Covariance Matrix of the discrete Markov Noise

$$Q_d = \sigma_{BI}^2 [1 - e^{-\frac{2}{T_c}\tau_s}]$$

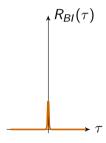
Autocorrelation of 1st order Markov





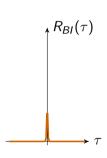
Small Correlation Time $T_c = 0.01$

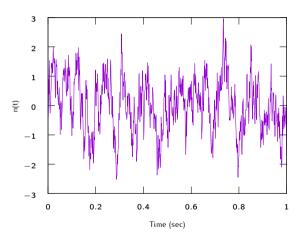




Small Correlation Time $T_c = 0.01$

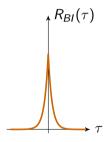






Larger Correlation Time $T_c = 0.1$





Larger Correlation Time $T_c = 0.1$



