

# Lecture

## INS/GPS Integration

EE 565: Position, Navigation, and Timing

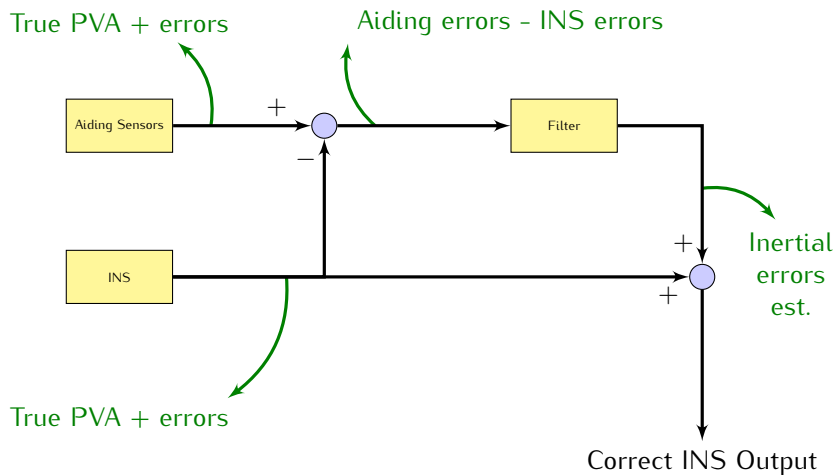
Lecture Notes Update on April 23, 2020

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### 1 Integration Architectures

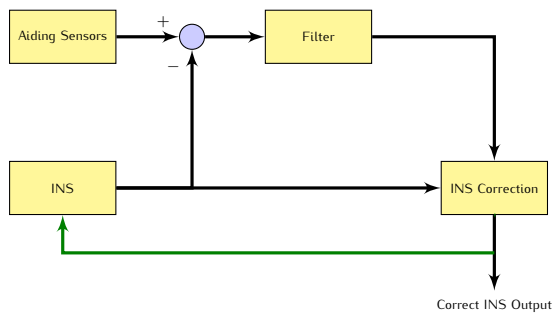
#### Open-Loop Integration



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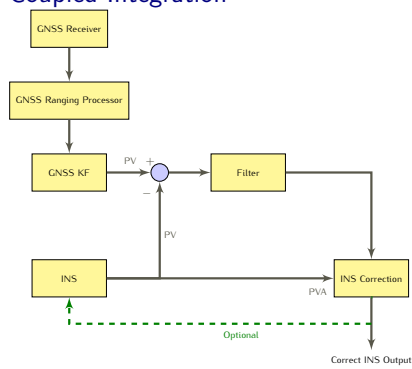
#### Closed-Loop Integration

If error estimates are fed back to correct the INS mechanization, a reset of the state estimates becomes necessary.



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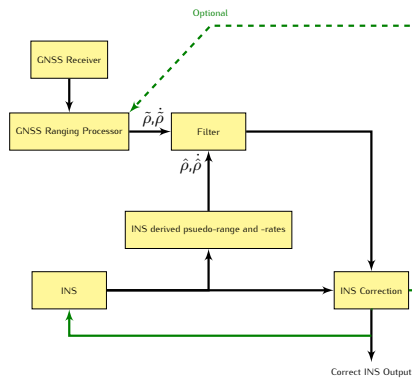
## Loosely Coupled Integration



- Simple
- Cascade KF therefore integration KF BW must be less than that of GNSS KF (e.g. update interval of 10s)
- Minimum of 4 satellites required

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## Tightly Coupled Integration



- No cascade KF
- KF BW must be kept less than the GNSS tracking loop
- Does not require 4 satellites

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## 2 Measurement Models

### ECEF INS/GNSS Loosely Coupled

$$\vec{z}_k^e = \begin{pmatrix} \hat{\vec{r}}_{GPS} - \hat{\vec{r}}_{eb}^e \\ \hat{\vec{v}}_{GPS} - \hat{\vec{v}}_{eb}^e \end{pmatrix} \quad (1)$$

$$\mathbf{H} = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} & \mathbf{I}_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & \mathbf{I}_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{pmatrix} \quad (2)$$

Theoretically, the lever arm from the INS to the GNSS antenna needs to be included, but in practice, the coupling of the attitude errors and gyro biases into the measurement through the lever arm is weak.

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## INS Derived Psuedo-Range and -Rates

The computed INS psuedo-range and rates with respect to the  $j$ th satellite,  $\hat{\rho}_j$  and  $\dot{\hat{\rho}}_j$

$$\hat{\rho}_j = \sqrt{[\hat{\mathbf{r}}_{es,j}^e - \hat{\mathbf{r}}_{eb}^e]^T [\hat{\mathbf{r}}_{es,j}^e - \hat{\mathbf{r}}_{eb}^e]} + \delta\hat{\rho}_{rc,j} \quad (3)$$

$$\dot{\hat{\rho}}_j = \hat{\mathbf{u}}_j^T [\hat{\mathbf{v}}_{s,j}^e - \hat{\mathbf{v}}_{eb}^e]^T + \delta\dot{\hat{\rho}}_{rc,j} \quad (4)$$

where

$$\hat{\mathbf{u}}_j = \frac{\hat{\mathbf{r}}_{es,j}^e - \hat{\mathbf{r}}_{eb}^e}{\|\hat{\mathbf{r}}_{es,j}^e - \hat{\mathbf{r}}_{eb}^e\|} \quad (5)$$

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## ECEF INS/GNSS Tightly Coupled

Pseudo-ranges are used instead of XYZ.

$$\bar{\mathbf{z}} = \begin{pmatrix} \bar{\mathbf{z}}_\rho \\ \bar{\mathbf{z}}_\dot{\rho} \end{pmatrix} \quad (6)$$

where

$$\bar{\mathbf{z}}_\rho = (\rho_{gps,1} - \hat{\rho}_1, \rho_{gps,2} - \hat{\rho}_2, \dots, \rho_{gps,n} - \hat{\rho}_n) \quad (7)$$

$$\bar{\mathbf{z}}_\dot{\rho} = (\dot{\rho}_{gps,1} - \dot{\hat{\rho}}_1, \dot{\rho}_{gps,2} - \dot{\hat{\rho}}_2, \dots, \dot{\rho}_{gps,n} - \dot{\hat{\rho}}_n) \quad (8)$$

$$\bar{\mathbf{x}}(t) = \left( \delta\psi_{eb}^e \quad \delta\mathbf{v}_{eb}^e \quad \delta\mathbf{r}_{eb}^e \quad \mathbf{b}_a \quad \mathbf{b}_g \quad \delta\rho_{rc} \quad \delta\dot{\rho}_{rc} \right)^T \quad (9)$$

$\rho_{gps,j}$ , and  $\dot{\rho}_{gps,j}$  and  $\hat{\rho}_j$ , and  $\dot{\hat{\rho}}_j$  are the psuedo-ranges and rates obtained from the GNSS and INS, respectively, for the  $j$ th satellite. These equations are none linear and an EKF needs to be used.  $\delta\rho_{rc}$  and  $\delta\dot{\rho}_{rc}$  are the clock bias and drift.

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## Extended Kalman Filter (EKF)

$$\mathbf{F}_k = \left. \frac{\partial \mathbf{f}(\bar{\mathbf{x}})}{\partial \bar{\mathbf{x}}} \right|_{\bar{\mathbf{x}}=\hat{\bar{\mathbf{x}}}_{k|k-1}}, \quad \mathbf{H}_k = \left. \frac{\partial \mathbf{h}(\bar{\mathbf{x}})}{\partial \bar{\mathbf{x}}} \right|_{\bar{\mathbf{x}}=\hat{\bar{\mathbf{x}}}_{k|k-1}} \quad (10)$$

where

$$\frac{\partial \mathbf{f}(\bar{\mathbf{x}})}{\partial \bar{\mathbf{x}}} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots \\ \vdots & \ddots & \vdots \end{pmatrix}, \quad \frac{\partial \mathbf{h}(\bar{\mathbf{x}})}{\partial \bar{\mathbf{x}}} = \begin{pmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \dots \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \dots \\ \vdots & \ddots & \vdots \end{pmatrix} \quad (11)$$

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## Tightly Coupled Linearized Measurement Matrix

$$\mathbf{H} = \begin{pmatrix} 0_{1 \times 3} & 0_{1 \times 3} & \hat{\mathbf{u}}_1^T & 0_{1 \times 3} & 0_{1 \times 3} & 1 & 0_{1 \times 3} \\ 0_{1 \times 3} & 0_{1 \times 3} & \hat{\mathbf{u}}_2^T & 0_{1 \times 3} & 0_{1 \times 3} & 1 & 0_{1 \times 3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1 \times 3} & 0_{1 \times 3} & \hat{\mathbf{u}}_n^T & 0_{1 \times 3} & 0_{1 \times 3} & 1 & 0_{1 \times 3} \\ 0_{1 \times 3} & \hat{\mathbf{u}}_1^T & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 1 \\ 0_{1 \times 3} & \hat{\mathbf{u}}_2^T & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1 \times 3} & \hat{\mathbf{u}}_n^T & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 1 \end{pmatrix} \quad (12)$$

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## Deeply Coupled

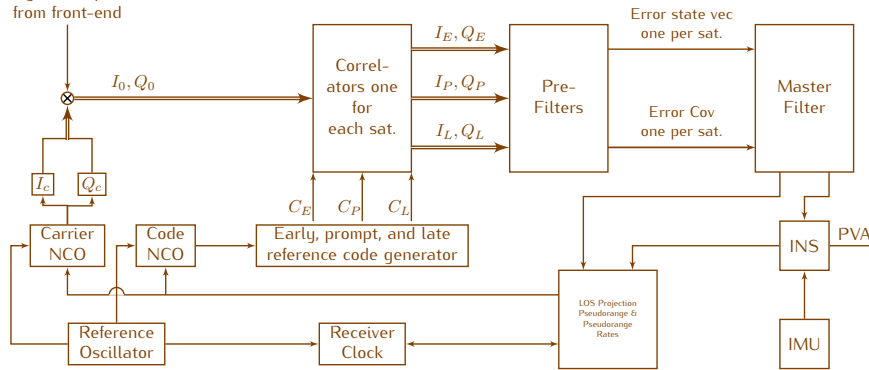
In deeply coupled GPS/INS integration

- the integration architecture utilizes the INS to aid in the tracking loops,
- allows fast acquisition,
- performance improvement with low signal-to-noise environments, and
- more resistance to jamming.

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## Deeply Coupled

Signal samples from front-end



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## 3 Other Considerations

### Observability

Are the states observable given a certain set of measurements?

The system is observable if the observability matrix

$$\mathcal{O}(k) = \begin{bmatrix} \mathbf{H}(k-n+1) \\ \mathbf{H}(k-n-2)\Phi(k-n+1) \\ \vdots \\ \mathbf{H}(k)\Phi(k-1) \dots \Phi(k-n+1) \end{bmatrix} \quad (13)$$

where  $n$  is the number of states, has a rank of  $n$ . The rank of  $\mathcal{O}$  is a binary indicator and does **not** provide a measure of how close the system is to being unobservable, hence, is prone to numerical ill-conditioning.

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### A Better Observability Measure

In addition to the computation of the rank of  $\mathcal{O}(k)$ , compute the Singular Value Decomposition (SVD) of  $\mathcal{O}(k)$  as

$$\mathcal{O} = U\Sigma V^* \quad (14)$$

and observe the diagonal values of the matrix  $\Sigma$ . Using this approach it is possible to monitor the variations in the system observability due to changes in system dynamics.

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### Corrupt/Missing Data

In Many cases the data from the aiding sensor may get corrupt and even not available. In those cases we can't use those measurements in the Kalman filter. Therefore

1. Set the Kalman gain to zero, or
2. Do not run the state or error covariance update steps.

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