EE 565: Position, Navigation and Timing

Navigation Equations: An Overview

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  - Using inertial sensors (accels & gyros) and an initial position and orientation, determine the vehicle's (i.e., body frame) current position, velocity, and attitude (PVA)

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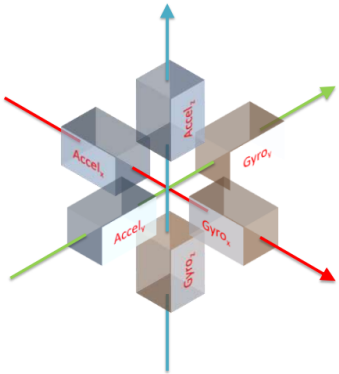
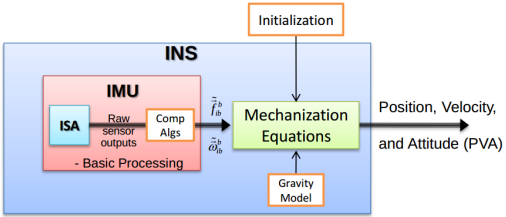
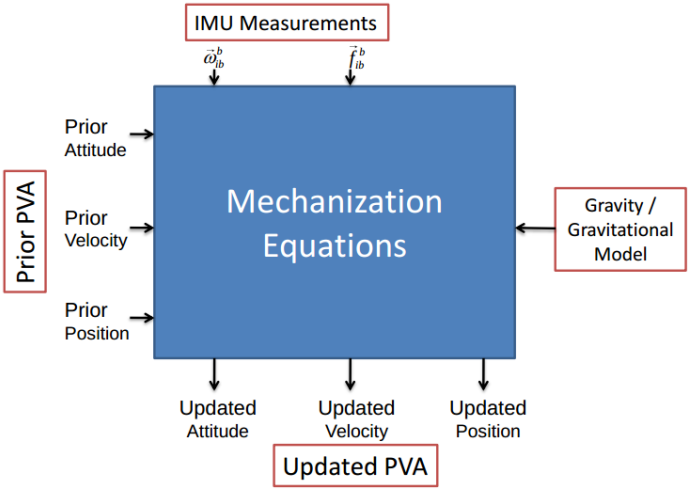
  - Where am I? Current PVA?
    - With respect to which frame?

- The process of "integrating" angular velocity & acceleration to determine one's position, velocity, and attitude (PVA)
- To measure the acceleration and angular velocity vectors we need at least 3-gyros and 3-accels
  - Typically configured in an orthogonal triad
- The "mechanization" can be performed *wrt*:
  - the ECI frame,
  - the ECEF frame,
  - the Nav frame, or
  - the tangential frame.



- An Inertial Navigation System (INS)
  - ISA — Inertial Sensor Assembly
    - Typically, 3-gyros, 3-accels, and basic electronics
  - IMU — Inertial Measurement Unit
    - ISA + compensation algorithms (i.e., basic processing)
  - INS — Inertial Navigation System
    - IMU + gravity model + "mechanization" algorithm




IMU Measurements
$\vec{\omega}_{ib}^{b}$
$\vec{f}_{ib}^{b}$
Prior Attitude
Prior PVA
Prior Velocity
Prior Position
Mechanization Equations
Gravity / Gravitational Model
Updated Attitude
Updated Velocity
Updated Position
Updated PVA

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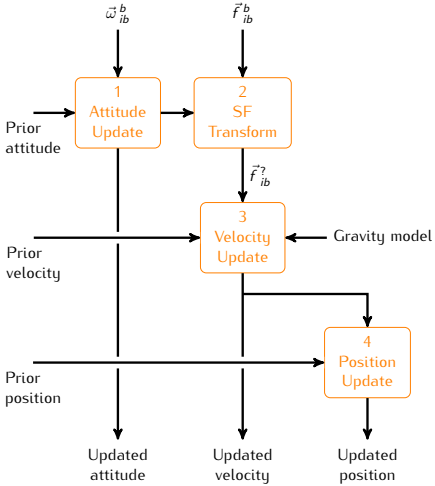
4. Update the position
   - integrate the result from step 3


$\vec{\omega}_{ib}^{\,b}$
$\vec{f}_{ib}^{\,b}$
1
Attitude
Update
2
SF
Transform
Prior
attitude
$\vec{f}_{ib}^{\,?}$
3
Velocity
Update
Gravity model
Prior
velocity
4
Position
Update
Prior
position
Updated
attitude
Updated
velocity
Updated
position