

Lecture

Gyro and Accel Noise Characteristics

EE 565: Position, Navigation and Timing

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Aly El-Osery and Kevin Wedeward, Electrical Engineering Dept., New Mexico Tech
In collaboration with
Stephen Bruder, Electrical & Computer Engineering, Embry-Riddle Aeronautical University

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1 Inertial Sensors Errors

Inertial Sensors — Sensor Models

- Accelerometer model

$$\tilde{\vec{f}}_{ib}^b = \vec{f}_{ib}^b + \Delta \vec{f}_{ib}^b = \vec{b}_a + (\mathcal{I} + M_a) \vec{f}_{ib}^b + \vec{w}_a \quad (1)$$

- Gyro Model

$$\tilde{\vec{\omega}}_{ib}^b = \vec{\omega}_{ib}^b + \Delta \vec{\omega}_{ib}^b = \vec{b}_g + (\mathcal{I} + M_g) \vec{\omega}_{ib}^b + G_g \vec{f}_{ib}^b + \vec{w}_g \quad (2)$$

- Typically, each measures along a single sense axis requiring three of each to measure the 3-tuple vector
- Bias errors are composite of fixed bias, bias instability, and bias stability

$$\vec{b} = b_{FB} + b_{BI} + b_{BS}$$

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2 Gyro Noise Characteristics

Gyro Constant Bias ($^{\circ}/h$)

A constant in the output of a gyro in the absence of rotation, in $^{\circ}/h$.

Error Growth

Linearly growing error in the angle domain of ϵt .

Model

Random constant.

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Gyro Integrated White Noise

Assuming the rectangular rule is used for integration, a sampling period of T_s and a time span of nT_s .

$$\int_0^t \epsilon(\tau) d\tau = T_s \sum_{i=1}^n \epsilon(t_i) \quad (3)$$

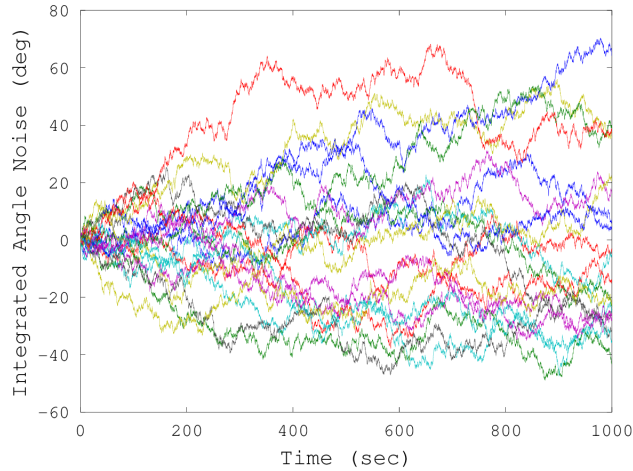
since $\mathbb{E}[\epsilon(t_i)] = 0$ and $Cov(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $Var[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E} \left[\int_0^t \epsilon(\tau) d\tau \right] = T_s n \mathbb{E}[\epsilon(t_i)] = 0, \forall i \quad (4)$$

$$Var \left[\int_0^t \epsilon(\tau) d\tau \right] = T_s^2 n Var[\epsilon(t_i)] = T_s t \sigma^2, \forall i \quad (5)$$

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Gyro Integrated White Noise



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Angle Random Walk ($^{\circ}/\sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_{\theta} = \sigma \sqrt{T_s t} \quad (6)$$

We define *ARW* as

$$ARW = \sigma_{\theta}(1) \quad (^{\circ}/\sqrt{h}) \quad (7)$$

In terms of PSD

$$ARW(^{\circ}/\sqrt{h}) = \frac{1}{60} \sqrt{PSD((^{\circ}/h)^2/Hz)} \quad (8)$$

Error Growth

ARW times root of the time in hours.

Model

White noise.

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Gyro Bias Instability ($^{\circ}/h$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

Error Growth

Variance grows over time.

Model

First order Gauss-Markov.

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3 Accel Noise Characteristics

Accel Constant Bias (μg)

A constant deviation in the accelerometer from the true value, in m/s^2 .

Error growth

Double integrating a constant bias error of ϵ results in a quadratically growing error in position of $\epsilon t^2/2$.

Model

Random constant.

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Velocity Random Walk ($m/s/\sqrt{h}$)

Integrating accelerometer output containing white noise results in velocity random walk (VRW) ($m/s/\sqrt{h}$). Similar to development of ARW, if we double integrate white noise we get

$$\iint_0^t \epsilon(\tau) d\tau d\tau = T_{s,sensor}^2 \sum_{i=1}^n \sum_{j=1}^i \epsilon(t_j) \quad (9)$$

Error Growth

Computing the standard deviation results in

$$\sigma_p \approx \sigma t^{(3/2)} \sqrt{\frac{T_s}{3}} \quad (10)$$

Model

White noise.

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Accel Bias Instability (μg)

Error growth

Grows as $t^{5/2}$.

Model

First order Gauss-Markov.

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4 Allan Variance

Allan Variance Introduction

It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966.

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Allan Variance Computation

1. Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \dots, M \leq N/2$.
2. For every n generate the sequence

$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \dots + x_{nj+n-1}}{n}, \quad j = 0, 1, \dots, \left\lfloor \frac{N}{n} \right\rfloor - 1 \quad (11)$$

3. Plot log-log of the Allan deviation which is square root of

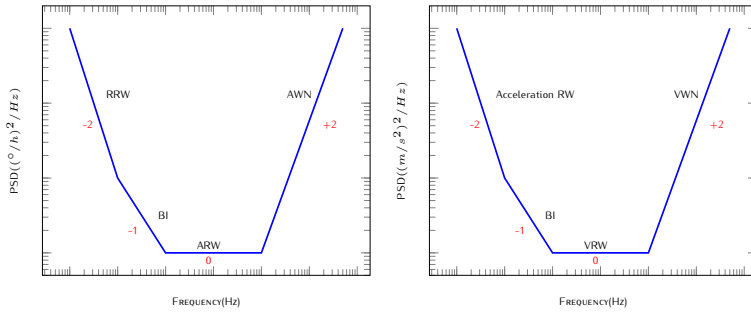
$$\sigma_{Allan}^2(nT_s) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_j - y_{j-1})^2 \quad (12)$$

versus averaging time $\tau = nT_s$

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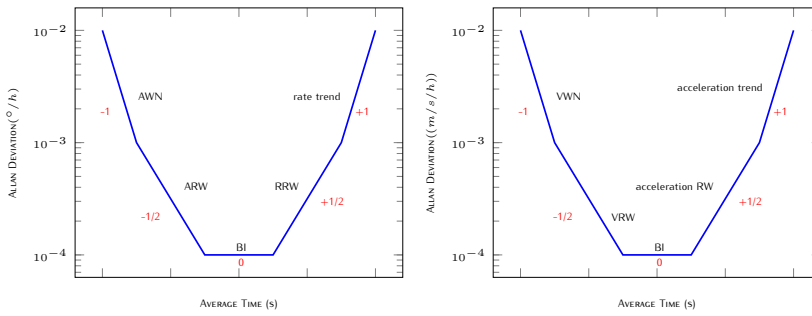
5 Using PSD and Allan Variance

One-sided PSD - Typical Slopes



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Allan Deviation - Typical Slopes



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Noise Parameters

Noise Type	AV $\sigma^2(\tau)$	PSD (2-sided)
Quantization Noise	$3\frac{\alpha^2}{\tau^2}$	$(2\pi f)^2\alpha^2T_s$
Angle/Velocity Random Walk	$\frac{\alpha^2}{\tau}$	α^2
Flicker Noise	$\frac{2\alpha^2 \ln(2)}{\pi}$	$\frac{\alpha^2}{2\pi f}$
Angular Rate/Accel Random Walk	$\frac{\alpha^2 \tau}{3}$	$\frac{\alpha^2}{(2\pi f)^2}$
Ramp Noise	$\frac{\alpha^2 \tau^2}{2}$	$\frac{\alpha^2}{(2\pi f)^3}$

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