EE 565: Position, Navigation and Timing Gyro and Accel Noise Characteristics

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March 29, 2023

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• Accelerometer model

$$\tilde{\vec{f}}_{ib}^{\ b} = \vec{f}_{ib}^{\ b} + \Delta \vec{f}_{ib}^{\ b} = \vec{b}_{a} + (\mathcal{I} + M_{a})\vec{f}_{ib}^{\ b} + \vec{w}_{a}$$
(1)

Gyro Model

$$\tilde{\vec{\omega}}_{ib}^{\ b} = \vec{\omega}_{ib}^{\ b} + \Delta \vec{\omega}_{ib}^{\ b} = \vec{b}_g + (\mathcal{I} + M_g) \vec{\omega}_{ib}^{\ b} + G_g \vec{f}_{ib}^{\ b} + \vec{w}_g$$
(2)

- Typically, each measures along a signle sense axis requiring three of each to measure the 3-tupple vector
- Bias errors are composite of fixed bias, bias instability, and bias stability

$$b = b_{FB} + b_{BI} + b_{BS}$$

Inertial Sensors Errors				
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A constant in the output of a gyro in the absence of rotation, in $^{\circ}/h$.

	Gyro Noise Character ●0000	istics Accel Noise Characteristics 000		
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A constant in the output of a gyro in the absence of rotation, in $^{\circ}/h$.

Error Growth

Linearly growing error in the angle domain of ϵt .

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Model

Random constant.

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Assuming the rectangular rule is used for integration, a sampling period of T_s and a time span of nT_s .

$$\int_0^t \epsilon(\tau) d\tau = T_s \sum_{i=1}^n \epsilon(t_i)$$
(3)

since $\mathbb{E}[\epsilon(t_i)] = 0$ and $Cov(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $Var[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E}\left[\int_{0}^{t} \epsilon(\tau) d\tau\right] = T_{s} n \mathbb{E}[\epsilon(t_{i})] = 0, \forall i$$
(4)

$$Var\left[\int_{0}^{t} \epsilon(\tau) d\tau\right] = T_{s}^{2} n Var[\epsilon(t_{i})] = T_{s} t \sigma^{2}, \forall i$$
(5)

 Inertial Sensors Errors
 Gyro Noise Characteristics
 Accel Noise Characteristics
 Allan Variance
 Using PSD and Allan Variance

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Angle Random Walk (°/ \sqrt{h})



(8)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_{\theta} = \sigma \sqrt{T_s t} \tag{6}$$

We define ARW as

$$ARW = \sigma_{\theta}(1) \qquad (^{\circ}/\sqrt{h}) \tag{7}$$

In terms of PSD

$$ARW(^{\circ}/\sqrt{h}) = rac{1}{60}\sqrt{PSD((^{\circ}/h)^2/Hz)}$$

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ARW times root of the time in hours.

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Model

White noise.

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- Due to flicker noise with spectrum 1/F.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

	Gyro Noise Characteristic	s Accel Noise Characteristics	Using PSD and Allan V	
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Variance grows over time.

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Error Growth

Variance grows over time.

Model

First order Gauss-Markov.

	Gyro Noise Characteris 00000	ics Accel Noise Characteristics 000		
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A constant deviation in the accelerometer from the true value, in m/s^2 .

		eristics Accel Noise Characteristics		
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Error growth

Double integrating a constant bias error of ϵ results in a quadratically growing error in position of $\epsilon t^2/2$.

		eristics Accel Noise Characteristics		
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Model

Random constant.

Velocity Random Walk $(m/s/\sqrt{h})$



Integrating accelerometer output containing white noise results in velocity random walk (VRW) $(m/s/\sqrt{h})$. Similar to development of ARW, if we double integrate white noise we get

$$\iint_{0}^{t} \epsilon(\tau) d\tau d\tau = T_{s,sensor}^{2} \sum_{i=1}^{n} \sum_{j=1}^{i} \epsilon(t_{j})$$
(9)

		ristics Accel Noise Characteristics		
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(9)

Error Growth

Computing the standard deviation results in

$$\sigma_{p} \approx \sigma t^{(3/2)} \sqrt{\frac{T_{s}}{3}} \tag{10}$$

		istics Accel Noise Characteristics		
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(10)

Model

White noise.

		tics Accel Noise Characteristics 0●0		
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Error growth

Grows as $t^{5/2}$.

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Accel Noise Characteristics

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Error growth

Grows as $t^{5/2}$.

Model

First order Gauss-Markov.

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It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966.

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Allan Variance Computation

- Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, ..., M \le N/2$.
- If a provide the sequence of the sequence o

$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \dots + x_{nj+n-1}}{n}, \quad j = 0, 1, \dots, \left[\frac{N}{n}\right] - 1 \tag{11}$$

Plot log-log of the Allan deviation which is square root of

$$\sigma_{Allan}^{2}(nT_{s}) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_{j} - y_{j-1})^{2}$$
(12)

versus averaging time $\tau = nT_s$

			Allan Variance ⊙●		
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One-sided PSD - Typical Slopes





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Allan Deviation - Typical Slopes





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Noise Type	AV $\sigma^2(\tau)$	PSD (2-sided)
Quantization Noise	$3\frac{\alpha^2}{\tau^2}$	$(2\pi f)^2 \alpha^2 T_s$
Angle/Velocity Random Walk	$\frac{\alpha^2}{\tau}$	α^2
Flicker Noise	$\frac{2\alpha^2\ln(2)}{\pi}$	$\frac{\alpha^2}{2\pi f}$
Angular Rate/Accel Random Walk	$\frac{\alpha^2 \tau}{3}$	$\frac{\alpha^2}{(2\pi f)^2}$
Ramp Noise	$\frac{\alpha^2 \tau^2}{2}$	$\frac{\alpha^2}{(2\pi f)^3}$

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