EE 565: Position, Navigation and Timing Sensor Technology

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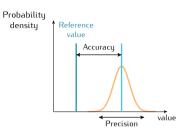
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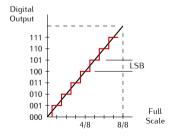
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Terminology



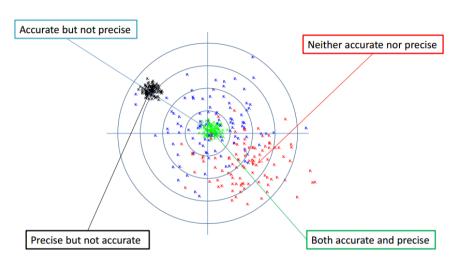
- **Accuracy:** Proximity of the measurement to the true value
- **Precision:** The consistency with which a measurement can be obtained
- **Resolution:** The magnitude of the smallest detectable change
- Sensitivitu: The ratio between the change in the output signal to a small change in input physical signal. Slope of the input-output fit line.
- Linearity: the deviation of the output from a "best" straight line fit for a given range of the sensor





Accuracy vs Precision





Inertial Sensors — Bias Errors



- Bias often the most critical error
 - Fixed Bias b_{FB}
 - Deterministic in nature and can be addressed by calibration
 - Often modeled as a function of temperature
 - Bias Stability b_{BS}
 - Varies from run-to-run as a random constant
 - Bias Instability b_{BI}
 - In-run bias drift typically modeled as random walk

Bias Errors

$$\Delta f = b_{a,FB} + b_{a,BI} + b_{a,BS} = b_a$$

$$\Delta \omega = b_{g,FB} + b_{g,BI} + b_{g,BS} = b_{g}$$

Inertial Sensors — Scale Factor Errors

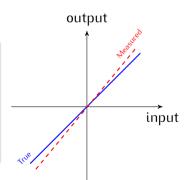


- Fixed scale factor error
 - Deterministic in nature and can be addressed by calibration
 - Often modeled as a function of temperature
- Scale Factor Stability s_a (accel) or s_g (gyro)
 - Varies from run-to-run as a random constant
 - Typically given in parts-per-million (ppm)

Scale Factor Errors

$$\Delta f = s_a f, \qquad \Delta \omega = s_g \omega$$

The scale factor represents a linear approximation to the steady-state sensor response over a given input range — True sensor response may have some non-linear characteristics



Inertial Sensors — Misalignment



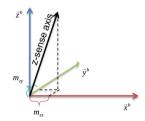
- Refers to the angular difference between the ideal sense axis alignment and true sense axis vector
 - a deterministic quantity typically given in milliradians

$$\Delta f_z = m_{a,zx} f_x + m_{a,zy} f_y$$

$$\Delta\omega_z=m_{g,z_X}\omega_x+m_{g,z_Y}\omega_y$$

• Combining misalignment & scale factor

$$\Delta \vec{f} = \begin{bmatrix} s_{a,x} & m_{a,xy} & m_{a,xz} \\ m_{a,yx} & s_{a,y} & m_{a,yz} \\ m_{a,zx} & m_{a,zy} & s_{a,z} \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = M_a \vec{f}_{ib}^b$$



Inertial Sensors — Cross-Axis Response



- Refers to the sensor output which occurs when the device is presented with a stimulus which is vectorially orthogonal to the sense axis
- Misalignment and cross-axis response are often difficult to distinguish Particularly during testing and calibration activities

Inertial Sensors — Other Noise Sources



Typically characterized as additive in nature

- May have a compound form
 - white noise
 - Guros: white noise in rate ⇒ Angle Random Walk (ARW)
 - Accels: white noise in accel ⇒ Velocity Random Walk (VRW)
 - Ouantization noise
 - May be due to LSB resolution in ADC's
 - Flicker noise
 - Colored noise

Inertial Sensors — Gyro Specific Errors



- G-sensitivitu
 - The gyro may be sensitive to acceleration
 - Primarily due to device mass assymetry
 - Mostly in Coriolis-based devices (MEMS)

$$\Delta \vec{\omega}_{ib}^{b} = G_{g} \vec{f}_{ib}^{b} a$$

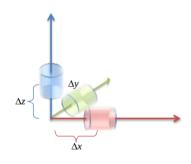
- G²-Sensitivitu
 - Anisoelastic effects
 - Due to products of orthogonal forces

Inertial Sensors — Accel Specific Errors



- Axis Offset
 - The accel may be mounted at a lever-arm distance from the "center" of the Inertial Measurement Unit (IMU)
 - Leads to an " $\omega^2 r$ " type effect

$$\Delta f_x = \omega_y^2 \Delta x + \omega_z^2 \Delta x = (\omega_y^2 + \omega_z^2) \Delta x$$



Inertial Sensors — Sensor Models



Accelerometer model

$$\tilde{\vec{f}}_{ib}^{\ b} = \vec{f}_{ib}^{\ b} + \Delta \vec{f}_{ib}^{\ b} = \vec{b}_a + (\mathcal{I} + M_a) \vec{f}_{ib}^{\ b} + \vec{w}_a \tag{1}$$

Gyro Model

$$\tilde{\vec{\omega}}_{ib}^{\ b} = \vec{\omega}_{ib}^{\ b} + \Delta \vec{\omega}_{ib}^{\ b} = \vec{b}_g + (\mathcal{I} + M_g) \vec{\omega}_{ib}^{\ b} + G_g \vec{f}_{ib}^{\ b} + \vec{w}_g$$
 (2)

• Typically, each measures along a signle sense axis requiring three of each to measure the 3-tupple vector