EE 565: Position, Navigation, and Timing Power Spectral Density Estimation

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Sensors suffer from noise effects that can not be removed through calibration, consquently, we need to

- understand the nature of the noise
- be able to extract parameters from actual data
- develop models to mimic noise in simulation to provide performance capabilities



Truth Infinitely long.

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Truth	Practice
 Infinitely long. 	• Finite length.

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Estimate the distribution of power in a signal. Unfortunately, truth and what is practical cause a problem.

Truth

- Infinitely long.
- Continuous in time and value.



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Truth

- Infinitely long.
- Continuous in time and value.

Practice

- Finite length.
- Discrete in time and value.

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Truth

- Infinitely long.
- Continuous in time and value.
- Provides true distribution of power.

Practice

- Finite length.
- Discrete in time and value.

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Truth

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- Only approximation of distribution of power.



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Let's make it more interesting

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Random Signa

Signals and Noise

Discrete Signals and Sy Do Power Spectral Density

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Truth

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Let's make it more interesting

The signal is stochastic in nature.

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(1)

Assume the voltage across a resistor R is e(t) and is producing a current i(t). The instantaneous power per ohm is $p(t) = e(t)i(t)/R = i^2(t)$.

Total Energy

$$E = \lim_{T o \infty} \int_{-T}^{T} i^2(t) dt$$

Average Power

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} i^2(t) dt$$
⁽²⁾





Total Normalized Energy

$$E \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$
(3)

Normalized Power

$$P \triangleq \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$
(4)

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Correlation



(5)

For Energy Signals

$$\phi(au) = \int_{-\infty}^\infty x(t) x(t+ au) dt$$

For Power Signals

$$R(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) x(t+\tau) dt$$
(6)

For Periodic Signals

$$R(\tau) = \frac{1}{T_0} \int_{T_0} x(t) x(t+\tau) dt$$
(7)

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(9)

Rayleigh's Energy Theorem or Parseval's theorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(F)|^2 dF$$
(8)

Energy Spectral Density

$$G(F) riangleq |X(F)|^2$$

with units of *volts*²-*sec*² or, if considered on a per-ohm basis, *watts*-*sec*/Hz=*joules*/Hz





$$P = \int_{-\infty}^{\infty} S(F) dF = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$
(10)

where we define S(F) as the power spectral density with units of watts/Hz.

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- Define an *experiment* with random *outcome*.
- Mapping of the outcome to a variable \Rightarrow random variable.
- Mapping of the outcome to a function \Rightarrow random function.

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$$F_X(x) =$$
probability that $X \le x = P(X \le x)$ (11)

Describes the manner random variables take different values.

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Probability Density Function (pdf)



$$f_X(x) = \frac{dF_X(x)}{dx} \tag{12}$$

and

$$P(x_1 < X \le x_2) = F_X(x_2) - F_X(x_1) = \int_{x_1}^{x_2} f_X(x) dx$$
(13)



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If the random variable X takes a set of discrete values x_i with probability p_i , the pdf of X is expressed in terms of Dirac delta functions, i.e.,

$$f_X(x) = \sum_i p_i \delta(x - x_i) \tag{14}$$







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Histogram and Pdf of random samples



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Mean of a Discrete RV

$$\bar{X} = \mathbb{E}[X] = \sum_{j=1}^{M} x_j P_j \tag{16}$$

Mean of a Continuous RV

$$\bar{X} = \mathbb{E}[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$
(17)

Variance of a RV

$$\sigma_X^2 \triangleq \mathbb{E}\left\{ [X - \mathbb{E}(X)]^2 \right\} = \mathbb{E}[X^2] - \mathbb{E}^2[X]$$
(18)

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(19)

(20)

(21)

Given a two random variables X and Y.

Covariance

$$\mu_{XY} = \mathbb{E}\left\{ [X - ar{x}][Y - ar{Y}]
ight\} = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

Correlation Coefficient

$$p_{XY} = \frac{\mu_{XY}}{\sigma_X \sigma_Y}$$

Autocorrelation

$${\sf F}_X(au)={\mathbb E}[X(t)X(t+ au)]$$

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Terminology





Figure: Sample functions of a random process

- $X(t, \zeta_i)$: sample function.
- The governing experiment: random or stochastic process.
- All sample functions: ensemble.
- $X(t_j, \zeta)$: random variable.

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If the joint pdfs depend only on the time difference regardless of the time origin, then the random process is known as *stationary*.

For stationary process means and variances are independent of time and the covariance depends only on the time difference.

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If the joint pdfs depends on the time difference but the mean and variances are time-independent, then the random process is known as *wide-sense-stationary*.

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If the time statistics equals ensemble statistics, then the random process is known as *ergodic*.

Any statistic calculated by averaging of all members of an ergodic ensemble at a fixed time can also be calculated by using a single representative waveform and averging over all time.

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Given a sample function $X(t, \zeta_i)$ of a random process, we obtain the power spectral density by

$$S(F) \stackrel{\mathcal{F}}{\longleftrightarrow} \Gamma(\tau)$$
 (22)

i.e., for a wide sense stationary signal, the power spectral density and autocorrelation are Fourier transform pairs.

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$$\xrightarrow{x(t)} H(F) \xrightarrow{y(t)}$$

$$S_Y(F) = |H(F)|^2 S_X(F)$$

Noise Shaping

If x(t) is white noise, we can design the filter h(t) to "shape" the noise.







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- Must sample more than twice bandwidth to avoid aliasing.
- FFT represents a periodic version of the time domain signal \rightarrow could have time domain aliasing.
- Number of points in FFT is the same as number of points in time domain signal.

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What we want is

$$\Gamma_X(\tau) = \mathbb{E}[X(t)X(t+\tau)] \xrightarrow{\mathcal{CTFT}} S_X(F)$$

For infinitely long signals.

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What we want is

$$\Gamma_X(\tau) = \mathbb{E}[X(t)X(t+\tau)] \xrightarrow{\mathcal{CTFT}} S_X(F)$$

For infinitely long signals.

What we can compute is

$$\gamma_X(m) = \mathbb{E}[X(n)X(n+m)] \xrightarrow{\mathcal{DFT}} P_X(f)$$

For finite length signals.

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As $N \to \infty$ and in the mean squared sense

Unbiased

Asymptotically the mean of the estimate approaches the true power.

Variance

Variance of the estimate approaches zero.

Resulting in a consistent estimate of the power spectrum.

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Periodogram

computed using 1/N times the magnitude squared of the FFT

 $\lim_{N\to\infty}\mathbb{E}[P_X(f)]=S_X(f)$

 $\lim_{N\to\infty} var[P_X(f)] = S_X^2(f)$

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Periodogram

computed using 1/N times the magnitude squared of the FFT

$$\lim_{N\to\infty}\mathbb{E}[P_X(f)]=S_X(f)$$

$$\lim_{N\to\infty} var[P_X(f)] = S_X^2(f)$$

Welch Method

computed by segmenting the data (allowing overlaps), windowing the data in each segment then computing the average of the resultant priodogram

$$\mathbb{E}[P_X(f)] = \frac{1}{2\pi MU} S_X(f) \circledast W(f)$$
$$var[P_X(f)] \approx \frac{9}{8L} S_X^2(f)$$

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Assuming data length *N*, segment length *M*, Bartlett window, and 50% overlap

- FFT length = $M = 1.28/\Delta f = 1.28F_s/\Delta F$
- Resulting number of segments = $L = \frac{2N}{M}$
- Length of data collected in sec. = $\frac{1.28L}{2\Delta F}$

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[Pxx,f] = pwelch(x,window,noverlap,... nfft,fs,'range')

You can use [] in fields that you want the default to be used.

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Fs = 1000; x = sqrt(0.1*Fs)*randn(1,100000); [Pxx,f] = pwelch(x,1024,[],[],Fs,'onesided');

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Fs = 1000; x = sqrt(0.1*Fs)*randn(1,100000); [Pxx,f] = pwelch(x,1024,[],[],Fs,'onesided');



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Fs = 1000; x = sqrt(0.1*Fs)*randn(1,100000); [Pxx,f] = pwelch(x,1024,[],[],Fs,'onesided');





				Power Spectra	Density 000
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[Pxx,f] = pwelch(x,128,[],[],Fs,'onesided')

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[Pxx,f] = pwelch(x,128,[],[],Fs,'onesided')



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[Pxx,f] = pwelch(x,128,[],[],Fs,'onesided')



- Reduced window size.
- Variance is now smaller.

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Fs = 100; t = 0:1/Fs:5; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,1024,[],[],Fs,'onesided');

pwelch Function - cos + WGN signal



Fs = 100; t = 0:1/Fs:5; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,1024,[],[],Fs,'onesided');



Frequency (Hz)

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pwelch Function - cos + WGN signal



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- Window larger than length of data.
- Frequency components can't be resolved.
- Variance high.

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Fs = 100; t = 0:1/Fs:5; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,1024,[],4096,Fs,'onesided');

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pwelch Function - cos + WGN signal



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Frequency (Hz)

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pwelch Function - cos + WGN signal



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Fs = 100; t = 0:1/Fs:5; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,1024,[],4096,Fs,'onesided');



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pwelch Function - cos + WGN signal



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Frequency (Hz)

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pwelch Function - cos + WGN signal



Fs = 100; t = 0:1/Fs:5; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,128,[],4096,Fs,'onesided');



- Decreasing the window size decreases the variance.
- Still can't resolve the two frequencies.



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Fs = 100; t = 0:1/Fs:50; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,128,[],4096,Fs,'onesided');

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pwelch Function - cos + WGN signal



Fs = 100; t = 0:1/Fs:50; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,128,[],4096,Fs,'onesided');



Frequency (Hz)

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pwelch Function - cos + WGN signal



Fs = 100; t = 0:1/Fs:50; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,128,[],4096,Fs,'onesided');



- Length of data sequence must be increased.
- Still can't resolve the two frequencies as the window size is too small.

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Fs = 100; t = 0:1/Fs:50; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,256,[],4096,Fs,'onesided');

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pwelch Function - cos + WGN signal



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Frequency (Hz)

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pwelch Function - cos + WGN signal



Fs = 100; t = 0:1/Fs:50; x = cos(2*pi*10*t)+cos(2*pi*11*t)+... sqrt(0.1*Fs)*randn(1,length(t)); [Pxx,f] = pwelch(x,256,[],4096,Fs,'onesided');



• Now we can resolve the two frequencies.

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- The length of the data sequence determines the maximum resolution that can be observed.
- Increasing the window length of each segment in the data increases the resolution.
- Decreasing the window length of each segment in the data decreases the variance of the estimate.
- nFFT only affects the amount of details shown and not the resolution.

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