EE 570: Homework 3

1. Consider the time-varying coordinate transformation matrix C_b^n given below that describes the orientation of the body frame as it rotates with respect to the navigation frame.

$$C_b^n = \begin{bmatrix} \cos(t) & \sin(t)\sin(t^2) & \sin(t)\cos(t^2) \\ 0 & \cos(t^2) & -\sin(t^2) \\ -\sin(t) & \cos(t)\sin(t^2) & \cos(t)\cos(t^2) \end{bmatrix}$$

   (a) Determine the analytic form of the time-derivative C_b^n (i.e., $\dot{C}_b^n = \frac{dC_b^n}{dt}$) via a term-by-term differentiation. Develop MATLAB functions which accept "t" (i.e., time) as a numerical input and return C_b^n and $\dot{C}_b^n$, respectively, as numerical outputs.

   (b) Using the C_b^n and $\dot{C}_b^n$ functions from above, compute the angular velocity vector ω_{nb}^n at time $t = 0$ sec (**Hint**: you might want to compute Ω_{nb}^n first).

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^{\,n}$) has the instantaneous rotation occurred?

   (c) Using C_b^n and $\dot{C}_b^n$ from above, compute the angular velocity vector $\vec{\omega}_{nb}^{\,n}$ at time $t = 0.5$ sec.

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^{\,n}$) has the instantaneous rotation occurred?

   (d) Using C_b^n and $\dot{C}_b^n$ from above, compute the angular velocity vector $\vec{\omega}_{nb}^{\,n}$ at time $t = 1$ sec.

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^{\,n}$) has the instantaneous rotation occurred?

   (e) In practice, direct measurement of the angular velocity vector $\vec{\omega}_{nb}^{\,n}$ can prove challenging, so a finite-difference approach may be taken given two sequential orientations represented by $C_b^n(t)$ and $C_b^n(t + \Delta t)$ a small time Δt apart. Consider the approximate value of the angular velocity vector $\vec{\omega}_{nb}^{\,n}$ derived by using the finite difference

$$\dot{C}_b^n(t) \approx \frac{C_b^n(t + \Delta t) - C_b^n(t)}{\Delta t}$$

   at times $t = 0, 0.5$, and 1 sec.

      i. Compare the "analytic" values for $\dot{\theta}$ and $\vec{k}_{nb}^{\,n}$ (found in parts b,c, &d) with your approximatins derived from the finite difference method using $\Delta t = 0.1$ sec. How large are the errors?

2. How small does the sampling time (i.e., Δt) need to be to get a "good" (better than 99.9%) approximation of the angular speed (i.e., $\dot{\theta}$)?