# Lecture
# Error Mechanization (NAV)
## EE 570: Location and Navigation

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## 1 Attitude

### NAV Attitude Error

$$\dot{C}_b^n = C_b^n(\Omega_{ib}^b - \Omega_{in}^b) = \frac{d}{dt}\left[(\mathcal{I} + [\delta\vec{\psi}_{nb}^{\,n}\times])\hat{C}_b^n\right] =$$

$$\begin{aligned}(\mathcal{I} + [\delta\vec{\psi}_{nb}^{\,n}\times])\hat{C}_b^n\Omega_{nb}^b &= [\delta\dot{\vec{\psi}}_{nb}^{\,n}\times]\hat{C}_b^n + (\mathcal{I} + [\delta\vec{\psi}_{nb}^{\,n}\times])\dot{\hat{C}}_b^n = \\ &\approx (\mathcal{I} + [\delta\vec{\psi}_{nb}^{\,n}\times])\hat{C}_b^n(\hat{\Omega}_{nb}^b + \delta\Omega_{ib}^b - \delta\Omega_{in}^b) \\ &\approx (\mathcal{I} + [\delta\vec{\psi}_{nb}^{\,n}\times])\hat{C}_b^n\hat{\Omega}_{nb}^b + \hat{C}_b^n(\delta\Omega_{ib}^b - \delta\Omega_{in}^b)\end{aligned}$$

$$[\delta\dot{\vec{\psi}}_{nb}^{\,n}\times] = \hat{C}_b^n(\delta\Omega_{nb}^b - \delta\Omega_{in}^b)\hat{C}_n^b = [\hat{C}_b^n(\delta\vec{\omega}_{ib}^{\,b} - \delta\vec{\omega}_{in}^{\,b})\times] \tag{1}$$

$$\delta\dot{\vec{\psi}}_{nb}^{\,n} = \hat{C}_b^n(\delta\omega_{ib}^b - \delta\vec{\omega}_{in}^{\,b}) \tag{2}$$

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### NAV Attitude Error (cont.) Computing $\delta\vec{\omega}_{in}^{\,b}$

Recall that

$$\vec{\omega}_{in}^{\,b} = C_n^b\vec{\omega}_{in}^{\,n}$$

Expressing the above equation in terms of estimates we get

$$\begin{aligned}\hat{\vec{\omega}}_{in}^b + \delta\vec{\omega}_{in}^{\,b} &= \hat{C}_n^b(\mathcal{I} - [\delta\vec{\psi}_{nb}^{\,n}\times])(\hat{\vec{\omega}}_{in}^n + \delta\vec{\omega}_{in}^{\,n}) \\ \delta\vec{\omega}_{in}^{\,b} &\approx \hat{C}_n^b(\delta\vec{\omega}_{in}^{\,n} - [\delta\vec{\psi}_{nb}^{\,n}\times]\hat{\vec{\omega}}_{in}^n)\end{aligned}$$

Substituting this result in Equation 2

$$\delta\dot{\vec{\psi}}_{nb}^{\,n} = -\hat{\Omega}_{in}^n\delta\vec{\psi}_{nb}^{\,n} + \hat{C}_b^n\delta\vec{\omega}_{ib}^{\,b} - \delta\vec{\omega}_{in}^{\,n} \tag{3}$$

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### NAV Attitude Error (cont.) Computing $\delta\vec{\omega}^{\,n}_{in}$

Using Taylor series and retaining the first order terms

$$\delta\vec{\omega}^{\,n}_{in} = \frac{\partial\hat{\vec{\omega}}^{n}_{in}}{\partial\hat{\vec{r}}^{\,n}_{eb}}\delta\vec{r}^{\,n}_{eb} + \frac{\partial\hat{\vec{\omega}}^{n}_{in}}{\partial\hat{\vec{v}}^{\,n}_{eb}}\delta\vec{v}^{\,n}_{eb}$$

where $\vec{r}^{\,n}_{eb} = [L_b, \lambda_b, h_b]^T$ and

$$\hat{\vec{\omega}}^{\,n}_{in} = \hat{\vec{\omega}}^{\,n}_{ie} + \hat{\vec{\omega}}^{\,n}_{en} = \begin{pmatrix} \omega_{ie}\cos\hat{L}_b + \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{R_E(\hat{L}_b)+\hat{h}_b} \\ -\frac{\hat{\vec{v}}^{\,n}_{eb,N}}{R_N(\hat{L}_b)+\hat{h}_b} \\ -\omega_{ie}\sin\hat{L}_b - \frac{\hat{\vec{v}}^{\,n}_{eb,E}\tan\hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} \end{pmatrix}$$



### NAV Attitude Error (cont.) Define terms for simplicity

$$-\hat{\Omega}^{\,n}_{in} = -[(\hat{\vec{\omega}}^{\,n}_{ie} + \hat{\vec{\omega}}^{\,n}_{en})\times] = \begin{pmatrix} 0 & -\omega_{ie}\sin\hat{L}_b - \frac{\hat{\vec{v}}^{\,n}_{eb,E}\tan\hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} & \frac{\hat{\vec{v}}^{\,n}_{eb,N}}{R_N(\hat{L}_b)+\hat{h}_b} \\ \omega_{ie}\sin\hat{L}_b + \frac{\hat{\vec{v}}^{\,n}_{eb,E}\tan\hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} & 0 & \omega_{ie}\cos\hat{L}_b + \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{R_E(\hat{L}_b)+\hat{h}_b} \\ -\frac{\hat{\vec{v}}^{\,n}_{eb,N}}{R_N(\hat{L}_b)+\hat{h}_b} & -\omega_{ie}\cos\hat{L}_b - \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{R_E(\hat{L}_b)+\hat{h}_b} & 0 \end{pmatrix} = F_{\psi\psi} \quad (4)$$

$$-\frac{\partial\hat{\vec{\omega}}^{\,n}_{in}}{\partial\hat{\vec{v}}^{\,n}_{eb}} = \begin{pmatrix} 0 & -\frac{1}{R_E(\hat{L}_b)+\hat{h}_b} & 0 \\ \frac{1}{R_N(\hat{L}_b)+\hat{h}_b} & 0 & 0 \\ 0 & \frac{\tan\hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} & 0 \end{pmatrix} = F_{\psi v} \quad (5)$$

$$-\frac{\partial\hat{\vec{\omega}}^{\,n}_{in}}{\partial\hat{\vec{r}}^{\,n}_{eb}} = \begin{pmatrix} \omega_{ie}\sin\hat{L}_b & 0 & \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{(R_E(\hat{L}_b)+\hat{h}_b)^2} \\ 0 & 0 & \frac{\hat{\vec{v}}^{\,n}_{eb,N}}{(R_N(\hat{L}_b)+\hat{h}_b)^2} \\ \omega_{ie}\cos\hat{L}_b + \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{(R_E(\hat{L}_b)+\hat{h}_b)\cos^2\hat{L}_b} & 0 & -\frac{\hat{\vec{v}}^{\,n}_{eb,E}\tan\hat{L}_b}{(R_E(\hat{L}_b)+\hat{h}_b)^2} \end{pmatrix} = F_{\psi r} \quad (6)$$



### NAV Attitude Error (cont.) Final Expression

$$\delta\dot{\vec{\psi}}^{\,n}_{nb} = F_{\psi\psi}\delta\vec{\psi}^{\,n}_{nb} + F_{\psi v}\delta\vec{v}^{\,n}_{eb} + F_{\psi r}\delta\vec{r}^{\,n}_{eb} + \hat{C}^n_b\delta\vec{\omega}^{\,b}_{ib} \quad (7)$$

where $F_{\psi\psi}$, $F_{\psi v}$ and $F_{\psi r}$ are defined by Equations 4, 5 and 6, respectively.



## 2 NAV Velocity Error

### Velocity

$$\dot{\vec{v}}^{\,n}_{eb} = C^n_b\vec{f}^{\,b}_{ib} + \vec{g}^{\,n}_b - (\Omega^n_{en} + 2\Omega^n_{ie})\vec{v}^{\,e}_{eb} \quad (8)$$

$$\begin{aligned} \dot{\hat{\vec{v}}}^{\,n}_{eb} &= \hat{C}^n_b\hat{\vec{f}}^{\,b}_{ib} + \hat{\vec{g}}^{\,n}_b - (\hat{\Omega}^n_{en} + 2\hat{\Omega}^n_{ie})\hat{\vec{v}}^{\,n}_{eb} \\ &= (\mathcal{I} - [\delta\vec{\psi}^{\,n}_{nb}\times])C^n_b(\vec{f}^{\,b}_{ib} - \delta\vec{f}^{\,b}_{ib}) + \hat{\vec{g}}^{\,n}_b - (\hat{\Omega}^n_{en} + 2\hat{\Omega}^n_{ie})\hat{\vec{v}}^{\,n}_{eb} \end{aligned} \quad (9)$$

Computing the $\delta\dot{\vec{v}}^{\,n}_{eb}$ we obtain

$$
\begin{aligned}
\delta\dot{\vec{v}}^{\,n}_{eb} &= \dot{\vec{v}}^{\,n}_{eb} - \dot{\hat{\vec{v}}}^{\,n}_{eb} \\
&= [\delta\vec{\psi}^{\,n}_{nb}\times]C^n_b\vec{f}^{\,b}_{ib} + \hat{C}^n_b\delta\vec{f}^{\,b}_{ib} + \delta\vec{g}^{\,n}_b + \\
&- (\hat{\Omega}^n_{en} + 2\hat{\Omega}^n_{ie})\delta\vec{v}^{\,n}_{eb} - (\delta\Omega^n_{en} + 2\delta\Omega^n_{ie})\hat{\vec{v}}^{\,n}_{eb}
\end{aligned}
$$

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**Velocity Cont.** $-(\hat{\Omega}^n_{en} + 2\hat{\Omega}^n_{ie})$

The term $-(\hat{\Omega}^n_{en} + 2\hat{\Omega}^n_{ie})$ is dervied as

$$
-[(\hat{\vec{\omega}}^{\,n}_{en} + 2\hat{\vec{\omega}}^{\,n}_{ie})\times] = \begin{pmatrix} 0 & -2\omega_{ie}\sin\hat{L}_b - \frac{\hat{\vec{v}}^{\,n}_{eb,E}\tan\hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} & \frac{\hat{\vec{v}}^{\,n}_{eb,N}}{R_N(\hat{L}_b)+\hat{h}_b} \\ 2\omega_{ie}\sin\hat{L}_b + \frac{\hat{\vec{v}}^{\,n}_{eb,E}\tan\hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} & 0 & 2\omega_{ie}\cos\hat{L}_b + \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{R_E(\hat{L}_b)+\hat{h}_b} \\ -\frac{\hat{\vec{v}}^{\,n}_{eb,N}}{R_N(\hat{L}_b)+\hat{h}_b} & -2\omega_{ie}\cos\hat{L}_b - \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{R_E(\hat{L}_b)+\hat{h}_b} & 0 \end{pmatrix} = F_{\Omega\Omega} \tag{10}
$$

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**Velocity Cont.** $-(\delta\hat{\Omega}^n_{en} + 2\delta\hat{\Omega}^n_{ie})\hat{\vec{v}}^{\,n}_{eb}$

The term $-(\delta\hat{\Omega}^n_{en} + 2\delta\hat{\Omega}^n_{ie})\hat{\vec{v}}^{\,n}_{eb}$ is dervied as

$$
-(\delta\hat{\Omega}^n_{en} + 2\delta\hat{\Omega}^n_{ie})\hat{\vec{v}}^{\,n}_{eb} = \frac{\partial F_{\Omega\Omega}}{\partial\hat{\vec{r}}^{\,n}_{eb}}\delta\vec{r}^{\,n}_{eb}\hat{\vec{v}}^{\,n}_{eb} + \frac{\partial F_{\Omega\Omega}}{\partial\hat{\vec{v}}^{\,n}_{eb}}\delta\vec{v}^{\,n}_{eb}\hat{\vec{v}}^{\,n}_{eb} \tag{11}
$$

.9

**Velocity Cont.** Gravity Error

$$
\delta\vec{g}^{\,n}_b \approx \frac{2g_0(\hat{L}_b)}{r^e_{eS}(\hat{L}_b)}\delta h_b \tag{12}
$$

.10

**Velocity Error Cont.** After tons of algebra

$$
\delta\dot{\vec{v}}^{\,n}_{eb} = F_{v\psi}\delta\vec{\psi}^{\,n}_{nb} + F_{vv}\delta\vec{v}^{\,n}_{eb} + F_{vr}\delta\vec{r}^{\,n}_{eb} + \hat{C}^n_b\delta\vec{f}^{\,b}_{ib} \tag{13}
$$

where $F_{v\psi}$, $F_{vv}$ and $F_{vr}$ are defined by Equations 14, 15 and 16, respectively. .11

**Velocity Cont.** $F_{v\psi}$, $F_{vv}$ and $F_{vr}$

$$
F_{v\psi} = -\left[(\hat{C}^n_b\hat{\vec{f}}^{\,b}_{ib})\times\right] \tag{14}
$$

$$
\mathbf{F}_{vv} = \begin{pmatrix} \frac{\hat{v}^n_{eb,D}}{(R_N(\hat{L}_b)+\hat{h}_b)^2} & -\frac{\hat{v}^n_{eb,E}\tan\hat{L}_b}{(R_E(\hat{L}_b)+\hat{h}_b)} - 2\omega_{ie}\sin\hat{L}_b & \frac{\hat{v}^n_{eb,N}}{R_N(\hat{L}_b)+\hat{h}_b} \\ \frac{\hat{v}^n_{eb,E}\tan\hat{L}_b}{(R_E(\hat{L}_b)+\hat{h}_b)} + 2\omega_{ie}\sin\hat{L}_b & \frac{\hat{v}^n_{eb,N}\tan\hat{L}_b + \hat{v}^n_{eb,D}}{R_N(\hat{L}_b)+\hat{h}_b} & \frac{\hat{v}^n_{eb,E}}{(R_E(\hat{L}_b)+\hat{h}_b)} + 2\omega_{ie}\cos\hat{L}_b \\ -\frac{2\hat{v}^n_{eb,N}}{R_N(\hat{L}_b)+\hat{h}_b} & -\frac{\hat{v}^n_{eb,E}}{(R_E(\hat{L}_b)+\hat{h}_b)} - 2\omega_{ie}\cos\hat{L}_b & 0 \end{pmatrix} \tag{15}
$$

$$\mathbf{F}_{vr} = \begin{pmatrix} -\frac{(\hat{v}^n_{eb,E})^2 \sec^2 \hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} - 2\hat{v}^n_{eb,E}\omega_{ie}\cos\hat{L}_b & 0 & \frac{(\hat{v}^n_{eb,E})^2 \tan\hat{L}_b}{(R_E(\hat{L}_b)+\hat{h}_b)^2} - \frac{\hat{v}^n_{eb,N}\hat{v}^n_{eb,D}}{(R_N(\hat{L}_b)+\hat{h}_b)^2} \\ \frac{\hat{v}^n_{eb,N}\hat{v}^n_{eb,E}\sec^2\hat{L}_b}{R_E(\hat{L}_b)+\hat{h}_b} + 2\hat{v}^n_{eb,N}\omega_{ie}\cos\hat{L}_b - 2\hat{v}^n_{eb,D}\omega_{ie}\sin\hat{L}_b & 0 & -\frac{\hat{v}^n_{eb,N}\hat{v}^n_{eb,E}\tan\hat{L}_b + \hat{v}^n_{eb,E}\hat{v}^n_{eb,D}}{(R_E(\hat{L}_b)+\hat{h}_b)^2} \\ 2\hat{v}^n_{eb,E}\omega_{ie}\sin\hat{L}_b & 0 & \left(\frac{(\hat{v}^n_{eb,E})^2}{(R_E(\hat{L}_b)+\hat{h}_b)^2} + \frac{(\hat{v}^n_{eb,N})^2}{(R_N(\hat{L}_b)+\hat{h}_b)^2} - \frac{2g_0(\hat{L}_b)}{r^e_{es}(\hat{L}_b}\right) \end{pmatrix} \tag{16}$$

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# 3 Nav Position Error

## Position

$$\dot{\hat{\vec{r}}}^{\,n}_{eb} = \begin{pmatrix} \frac{\hat{\vec{v}}^{\,n}_{eb,N}}{R_N(\hat{L}_b+\hat{h}_b} \\ \frac{\hat{\vec{v}}^{\,n}_{eb,E}}{\cos\hat{L}_b(R_E(\hat{L_b})+\hat{h}_b}) \\ -\hat{\vec{v}}^{\,n}_{eb_D} \end{pmatrix} \tag{17}$$

Computing $\delta\dot{\vec{r}}^{\,n}_{eb}$ using taylor series expansion and retaining only the first order terms

$$\delta\dot{\vec{r}}^{\,n}_{eb} = F_{r\psi}\delta\vec{\psi}^{\,n}_{nb} + F_{rv}\delta\vec{v}^{\,n}_{eb} + F_{rr}\delta\vec{r}^{\,n}_{eb} \tag{18}$$

where $F_{r\psi}$, $F_{rv}$ and $F_{rr}$ are defined by Equations 19, 20 and 21, respectively.

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## Position Cont. $F_{r\psi}$, $F_{rv}$ and $F_{rr}$

$$F_{r\psi} = 0_{3\times 3} \tag{19}$$

$$\mathbf{F}_{rv} = \begin{pmatrix} \frac{1}{R_N(\hat{L}_b)+\hat{h}_b} & 0 & 0 \\ 0 & \frac{1}{(R_E(\hat{L}_b)+\hat{h}_b)\cos\hat{L}_b} & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{20}$$

$$\mathbf{F}_{rr} = \begin{pmatrix} 0 & 0 & -\frac{\hat{v}^n_{eb,N}}{(R_N(\hat{L}_b)+\hat{h}_b)^2} \\ \frac{\hat{v}^n_{eb,E}}{(R_N(\hat{L}_b)+\hat{h}_b)\cos^2\hat{L}_b} & 0 & \frac{\hat{v}^n_{eb,E}}{(R_N(\hat{L}_b)+\hat{h}_b)^2\cos\hat{L}_b} \\ 0 & 0 & 0 \end{pmatrix} \tag{21}$$

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## 4 Summary

### Summary - in terms of $\delta\vec{f}^{\,b}_{ib}, \delta\vec{\omega}^{\,b}_{ib}$

$$\begin{pmatrix} \delta\dot{\vec{\psi}}^{\,n}_{nb} \\ \delta\dot{\vec{v}}^{\,n}_{nb} \\ \delta\dot{\vec{r}}^{\,n}_{nb} \end{pmatrix} = \begin{pmatrix} F_{\psi\psi} & F_{\psi v} & F_{\psi r} \\ F_{v\psi} & F_{vv} & F_{vr} \\ 0_{3\times 3} & F_{rv} & F_{rr} \end{pmatrix} \begin{pmatrix} \delta\vec{\psi}^{n}_{nb} \\ \delta\vec{v}^{n}_{nb} \\ \delta\vec{r}^{n}_{nb} \end{pmatrix} + \begin{pmatrix} 0 & \hat{C}^{n}_{b} \\ \hat{C}^{n}_{b} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta\vec{f}^{\,b}_{ib} \\ \delta\vec{\omega}^{\,b}_{ib} \end{pmatrix} \tag{22}$$

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## A Basic Definitions

### Notation Used

- *Truth value*
$$\vec{x}$$
- *Measured value*
$$\tilde{\vec{x}}$$
- *Estimated or computed value*
$$\hat{\vec{x}}$$
- *Error*
$$\delta\vec{x} = \vec{x} - \hat{\vec{x}}$$

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## B Linearization

### Linearization using Taylor Series Expansion

Given a non-linear system $\dot{\vec{x}} = f(\vec{x}, t)$

Let's assume we have an estimate of $\vec{x}$, i.e., $\hat{\vec{x}}$ such that $\vec{x} = \hat{\vec{x}} + \delta\vec{x}$

$$\dot{\vec{x}} = \dot{\hat{\vec{x}}} + \delta\dot{\vec{x}} = f(\hat{\vec{x}} + \delta\vec{x}, t) \tag{23}$$

Using Taylor series expansion

$$\begin{aligned} f(\hat{\vec{x}} + \delta\vec{x}, t) = \dot{\hat{\vec{x}}} + \delta\dot{\vec{x}} = \; & f(\hat{\vec{x}}, t) \; + \left.\frac{\partial f(\vec{x}, t)}{\partial \vec{x}}\right|_{\vec{x}=\hat{\vec{x}}} \delta\vec{x} + H.O.T \\ \approx \; & \dot{\hat{\vec{x}}} \; + \left.\frac{\partial f(\vec{x}, t)}{\partial \vec{x}}\right|_{\vec{x}=\hat{\vec{x}}} \delta\vec{x} \\ \Rightarrow \delta\dot{\vec{x}} \approx \; & \left.\frac{\partial f(\vec{x}, t)}{\partial \vec{x}}\right|_{\vec{x}=\hat{\vec{x}}} \delta\vec{x} \end{aligned} \tag{24}$$

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## C Inertial Measurements

### Actual Measurements

Initially the accelerometer and gyroscope measurements, $\tilde{\vec{f}}^{\,b}_{ib}$ and $\tilde{\vec{\omega}}^{\,b}_{ib}$, respectively, will be modeled as

$$\tilde{\vec{f}}^{\,b}_{ib} = \vec{f}^{\,b}_{ib} + \Delta\vec{f}^{\,b}_{ib} \tag{25}$$

$$\tilde{\vec{\omega}}^{\,b}_{ib} = \vec{\omega}^{\,b}_{ib} + \Delta\vec{\omega}^{\,b}_{ib} \tag{26}$$

where $\vec{f}^{\,b}_{ib}$ and $\vec{\omega}^{\,b}_{ib}$ are the specific force and angular rates, respectively; and $\Delta\vec{f}^{\,b}_{ib}$ and $\Delta\vec{\omega}^{\,b}_{ib}$ represents the errors. In later lectures we will discuss more detailed description of these errors.

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### Error Modeling Example

Accelerometers

$$\tilde{\vec{f}}^{\,b}_{ib} = \vec{b}_a + (\mathcal{I} + M_a)\vec{f}^{\,b}_{ib} + \vec{nl}_a + \vec{w}_a \tag{27}$$

Gyroscopes

$$\tilde{\vec{\omega}}^{\,b}_{ib} = \vec{b}_g + (\mathcal{I} + M_g)\vec{\omega}^{\,b}_{ib} + G_g\vec{f}^{\,b}_{ib} + \vec{w}_g \tag{28}$$

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### Pos, Vel, Force and Angular Rate Errors

- Position error

$$\delta\vec{r}^{\,\gamma}_{\beta b} = \vec{r}^{\,\gamma}_{\beta b} - \hat{\vec{r}}^{\,\gamma}_{\beta b} \tag{29}$$

- Velocity error

$$\delta\vec{v}^{\,\gamma}_{\beta b} = \vec{v}^{\,\gamma}_{\beta b} - \hat{\vec{v}}^{\,\gamma}_{\beta b} \tag{30}$$

- Specific force errors

$$\delta\vec{f}^{\,b}_{ib} = \vec{f}^{\,b}_{ib} - \hat{\vec{f}}^{\,b}_{ib} \tag{31}$$

$$\Delta_e\vec{f}^{\,b}_{ib} = \Delta\vec{f}^{\,b}_{ib} - \Delta\hat{\vec{f}}^{\,b}_{ib} = -\delta\vec{f}^{\,b}_{ib} \tag{32}$$

- Angular rate errors

$$\delta\vec{\omega}^{\,b}_{ib} = \vec{\omega}^{\,b}_{ib} - \hat{\vec{\omega}}^{\,b}_{ib} \tag{33}$$

$$\Delta_e\vec{\omega}^{\,b}_{ib} = \Delta\vec{\omega}^{\,b}_{ib} - \Delta\hat{\vec{\omega}}^{\,b}_{ib} = -\delta\vec{\omega}^{\,b}_{ib} \tag{34}$$

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## D Attitude Error

### Attitude Error Definition

Define

$$\delta C^{\gamma}_{b} = C^{\gamma}_{b}\hat{C}^{b}_{\gamma} = e^{[\delta\vec{\psi}^{\,\gamma}_{\gamma b}\times]} \approx \mathcal{I} + [\delta\vec{\psi}^{\,\gamma}_{\gamma b}\times] \tag{35}$$

This is the error in attitude resulting from errors in estimating the angular rates. .21

### Attitude Error Properties

The attitude error is a multiplicative small angle transformation from the actual frame to the computed frame

$$\hat{C}^{\gamma}_{b} = (\mathcal{I} - [\delta\vec{\psi}^{\,\gamma}_{\gamma b}\times])C^{\gamma}_{b} \tag{36}$$

Similarly,

$$C^{\gamma}_{b} = (\mathcal{I} + [\delta\vec{\psi}^{\,\gamma}_{\gamma b}\times])\hat{C}^{\gamma}_{b} \tag{37}$$

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# E Estimates of Sensor Measurements

### Specific Force and Agnular Rates

Similarly we can attempt to estimate the specific force and angular rate by applying correction based on our estimate of the error.

$$\hat{\vec{f}}^{\,b}_{ib} = \tilde{\vec{f}}^{\,b}_{ib} - \Delta\hat{\vec{f}}^{\,b}_{ib} \tag{38}$$

$$\hat{\vec{\omega}}^{\,b}_{ib} = \tilde{\vec{\omega}}^{\,b}_{ib} - \Delta\hat{\vec{\omega}}^{\,b}_{ib} \tag{39}$$

where $\hat{\vec{f}}^{\,b}_{ib}$ and $\hat{\vec{\omega}}^{\,b}_{ib}$ are the accelerometer and gyroscope estimated calibration values, respectively. .23