# Lecture

## Error Modeling

### EE 570: Location and Navigation

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.1

### Overview

Given a state space system

$$\dot{\vec{x}}(t) = F(t)\vec{x}(t) + G(t)\vec{w}(t) \tag{1}$$

and the measurement equation

$$\vec{y}(t) = H(t)\vec{x}(t) + \vec{v}(t) \tag{2}$$

If $\vec{w}$ and $\vec{v}$ are Gaussian random processes, then the system above is known as *Gauss-Markov process*. This process is used to describe a variety of error models of interest. Particularly,

- random constant,
- random walk, and
- first order Markov.

.2

### Random Constants

Assuming that the error is due to an unknown constant (i.e. random constant), the following model is used

$$\dot{\vec{x}}(t) = 0 \tag{3}$$

This model is useful in describing effects such as bias stability, scale factor errors, and misalignments. The value can vary from turn-on to turn-on.

.3

### Random Walk

An INS system requires the integration of accel and gyro measurements. This leads to the Gauss-Markov process

$$x(t) = \int_0^t w(\nu)d\nu \Rightarrow \dot{x} = w(t)$$

with

$$\mathbb{E}\{w(t)\} = 0 \tag{4}$$

$$\mathbb{E}\{w(t)w(t+\tau)\} = Q\delta(\tau) \tag{5}$$

$$\mathbb{E}\{x(t)\} = 0$$

$$\mathbb{E}\{x^2(t)\} = Qt$$

called random walk. .4

## Random Walk Cont.

- In the case of the gyroscope, $w$ would be in the rate domain and thus has units of $deg/s$ and the units of $x(t)$ is degrees.
- $\mathbb{E}\{x^2(t)\} = Qt$ has units of $deg^2$.
- Therefore, $Q$ has the units of $\frac{deg^2}{s} = \frac{(deg/s)^2}{Hz}$ which is the PSD level of the gyro white noise.
- This matches the units in Eq. 5 (Note that $\delta(\tau)$ has units of $1/\text{time}$).
- In simulation the power of the white noise $w$ that is experienced by the system is $F_s Q$, where $F_s$ is the sampling rate.
- Consquently, $\sigma_w = \sqrt{F_s Q}$ and has the units of $deg/s$
- Recall that the gyro angle random walk (ARW)

$$ARW(^\circ/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((^\circ/h)^2/Hz)} \tag{6}$$

.5

## First Order Markov Noise

Frequently used to model bias instability (BI)

**State Equation**

$$\dot{b}(t) = -\frac{1}{T_c} b(t) + w(t) \tag{7}$$

**Autocorrelation**

$$\mathbb{E}\{b(t)b(t+\tau)\} = \sigma_{BI}^2 e^{-|\tau|/T_c} \tag{8}$$

where

$$\mathbb{E}\{w(t)w(t+\tau)\} = Q\delta(t-\tau) \tag{9}$$

$$Q = \frac{2\sigma_{BI}^2}{T_c} \tag{10}$$

and $T_c$ is the correlation time. .6

## Discrete First Order Markov Noise

**State Equation**

$$b_k = e^{-\frac{1}{T_c}\tau_s} b_{k-1} + w_{k-1} \tag{11}$$

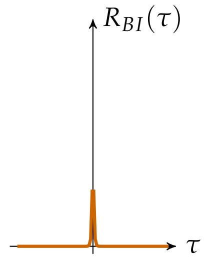
**Covariance Matrix of the discrete Markov Noise**

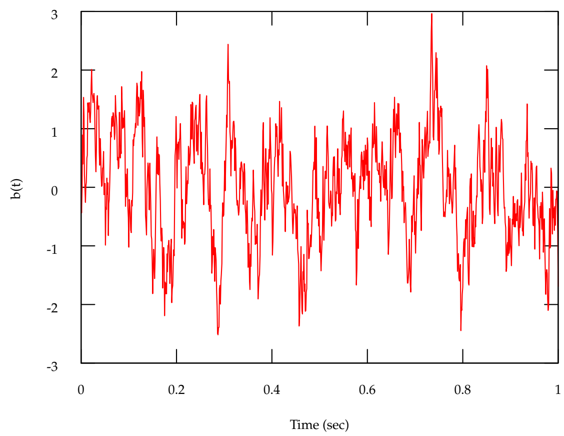
$$Q_d = \sigma_{BI}^2 [1 - e^{-\frac{2}{T_c}\tau_s}] \tag{12}$$

.7

## Autocorrelation of 1st order Markov .8

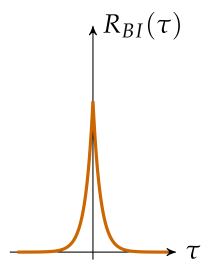
## Small Correlation Time $T_c = 0.01$

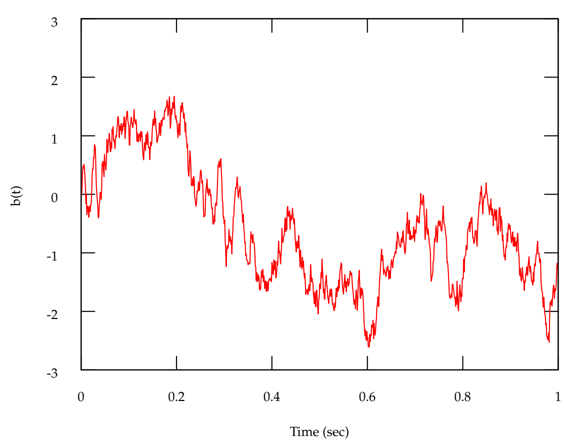



.9

## Larger Correlation Time $T_c = 0.1$




.10