EE 570: Location and Navigation

Navigation Mathematics: Kinematics (Coordinate Frames)

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Earth-Centered Inertial (ECI) Coordinate Frame

- An inertial coordinate frame is one that does **not** accelerate (rectilinearly) or change its orientation (*wrt* the "stars")
  - All inertial sensors measure "inertial" motion
    - Gyroscopes measure rate of change of inertial orientation
    - accelerometers measure inertial acceleration
- The ECI frame will be referred to as the *i*-frame

Coordinate Frame - ECI

- The origin of the ECI is located at the center of mass of the earth
- the z-axis points along the nominal axis of rotation of the earth
  - True north **not** magnetic north!!
- The x-axis lies in the equatorial plane and points from the earth to the sun at the vernal equinox
  - Defined by the intersection of the equatorial plane and the earth-sun orbital plane
- The y-axis is simply chosen to conform to a right hand coordinate system

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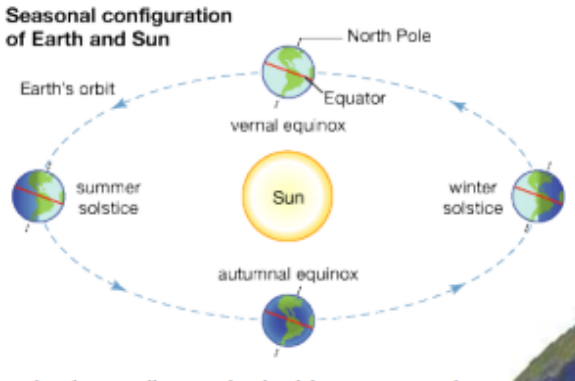
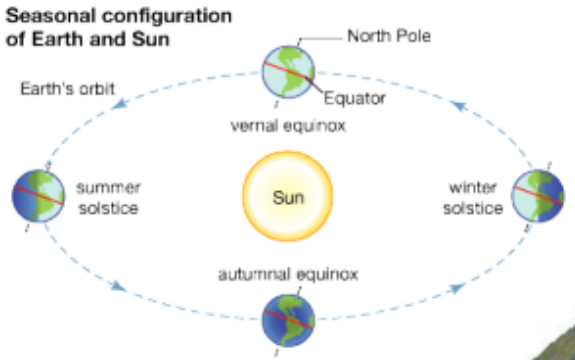
The ECI coordinate frame does **not** rotate with the earth

ECI Coordinate Frame



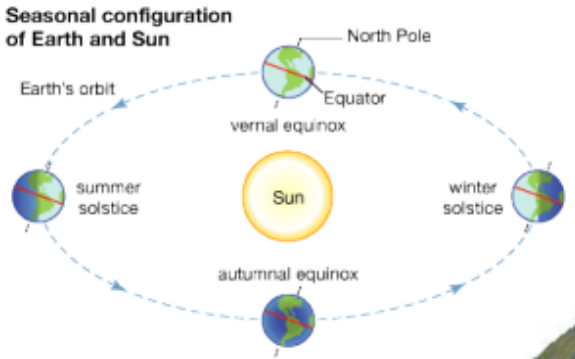
ECI Coordinate Frame

- z-axis points along the nominal earth axis of rotation



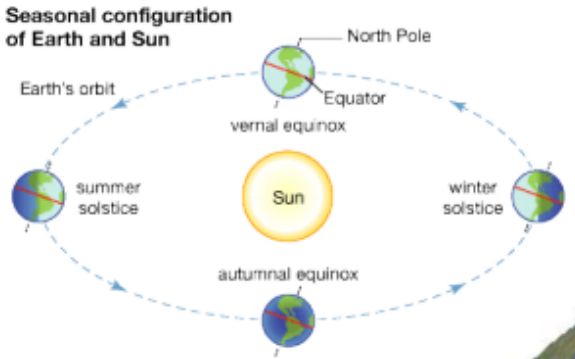
ECI Coordinate Frame

- z-axis points along the nominal earth axis of rotation
- x-axis points towards vernal (spring) equinox



ECI Coordinate Frame

- z-axis points along the nominal earth axis of rotation
- x-axis points towards vernal (spring) equinox
- y-axis completes a right hand coordinate system



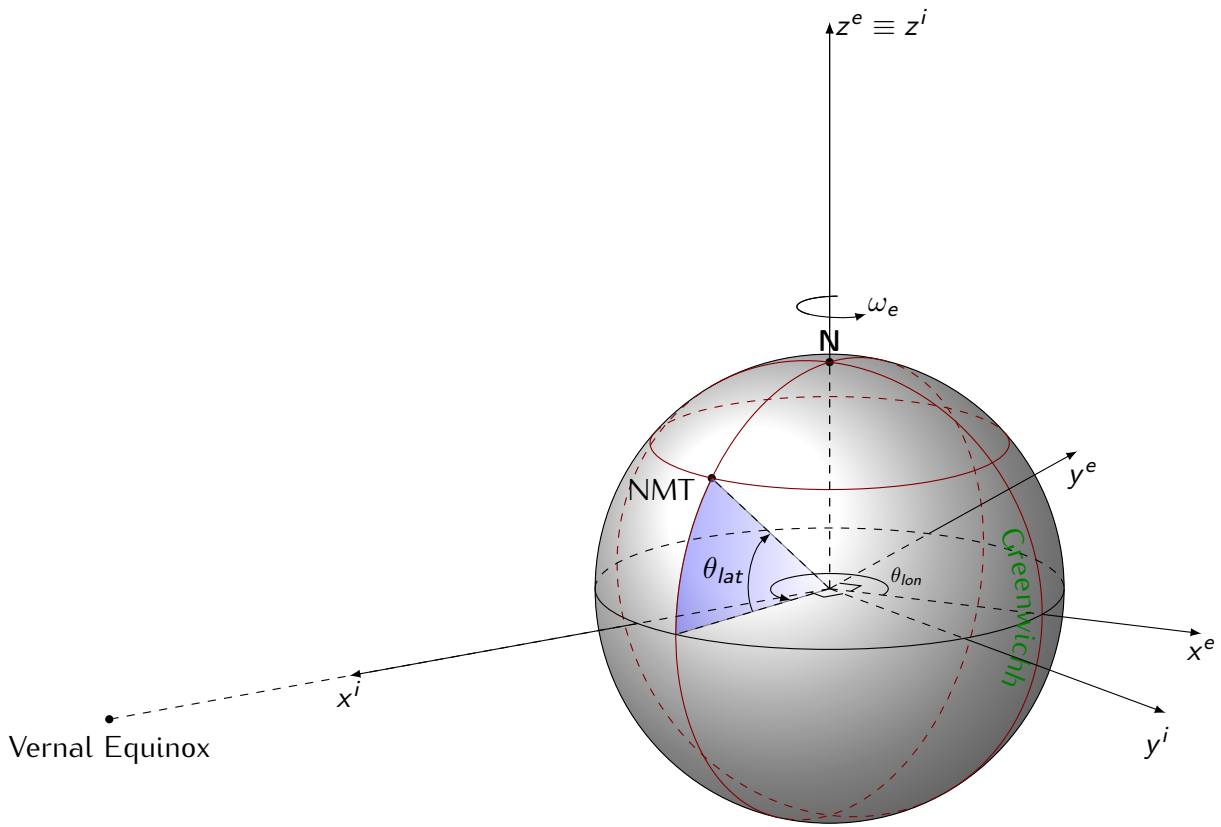
Earth-Centered Earth-Fixed (ECEF) Coordinate Frame

- The ECEF coordinate frame is **not** an inertial frame
- The ECEF coordinate frame is fixed with respect to the earth
- The ECEF coordinate frame will be referred to as the *e*-frame

ECEF Coordinate Frame

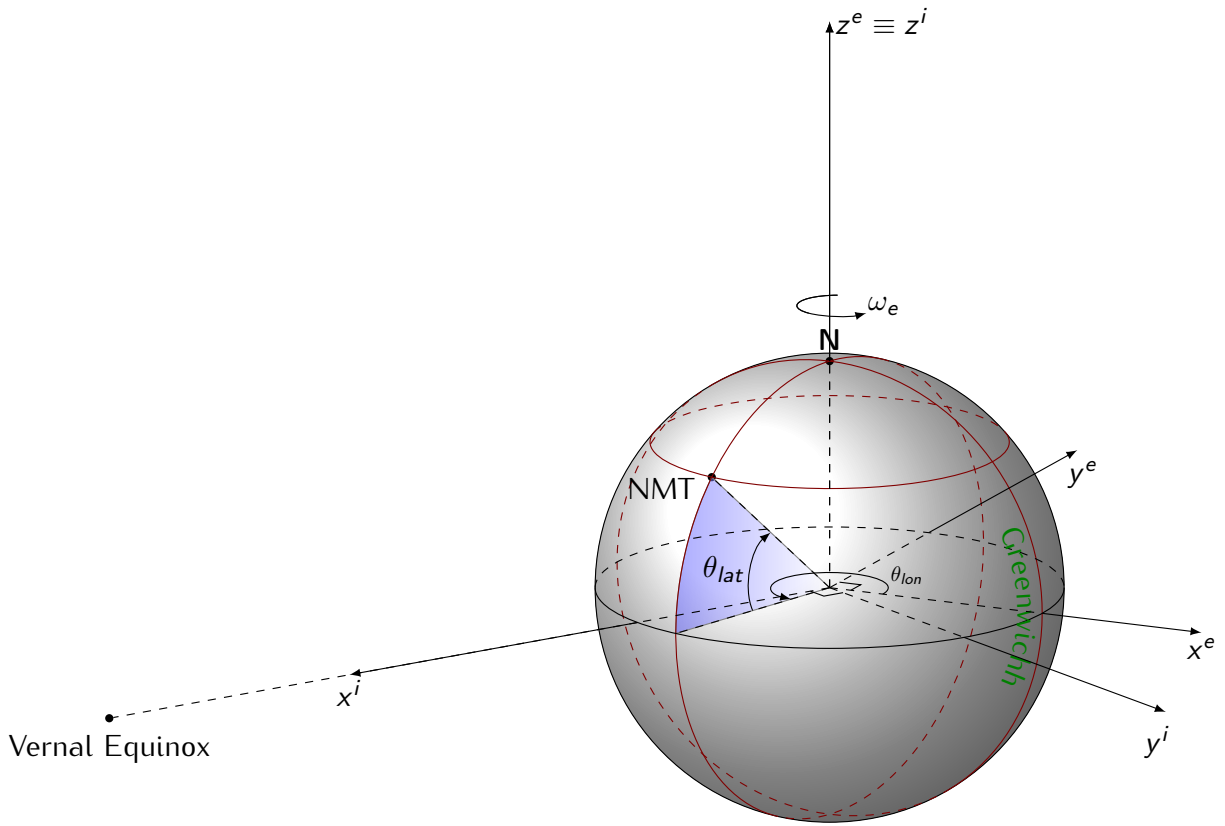
- The origin of the ECEF frame is located at the center of the mass of the earth (same as ECI)
- The z-axis points along the nominal axis of rotation of the earth (same as ECI)
- The x-axis lies at the intersection of the equatorial plane and the reference meridian plane (i.e., Greenwich meridian)
  - Concept of latitude and longitude
- The y-axis is simply chosen to conform to a right hand coordinate system

ECEF Coordinate Frame



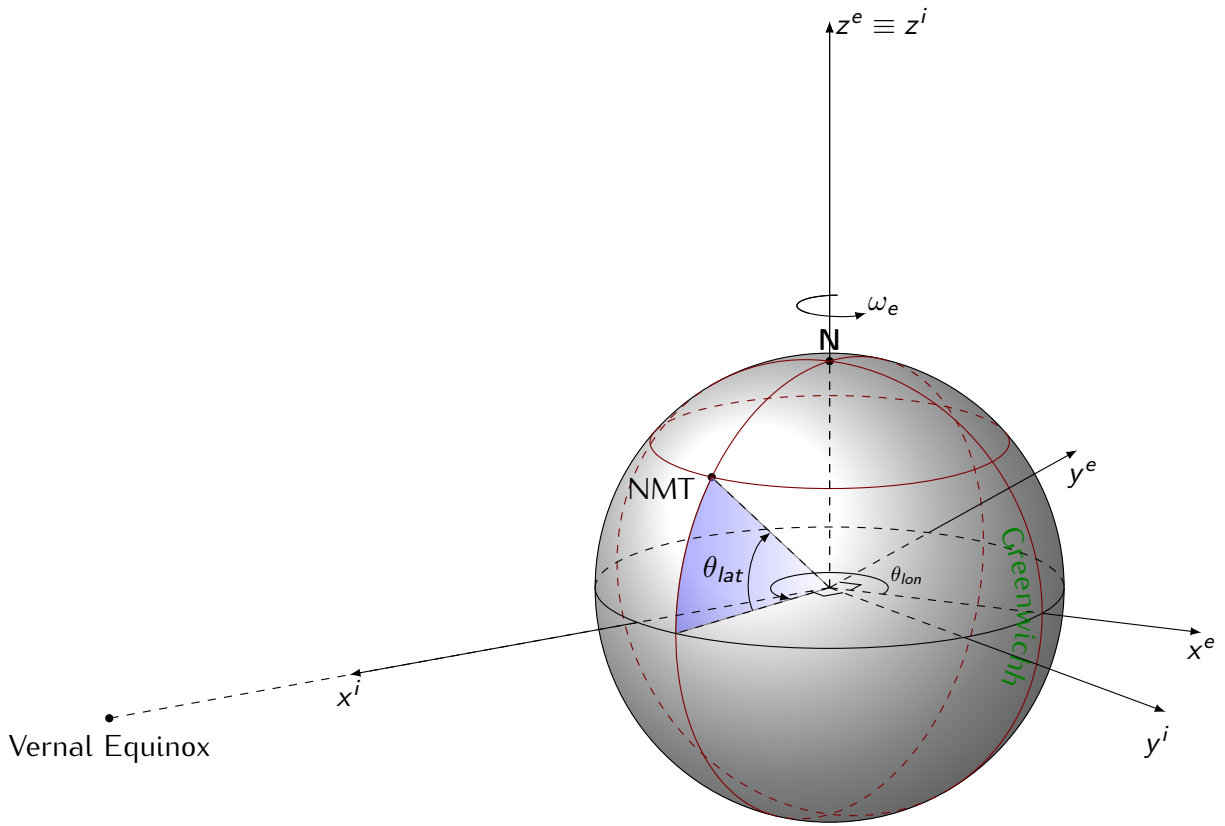
ECEF Coordinate Frame

- z-axis points along the nominal earth axis of rotation



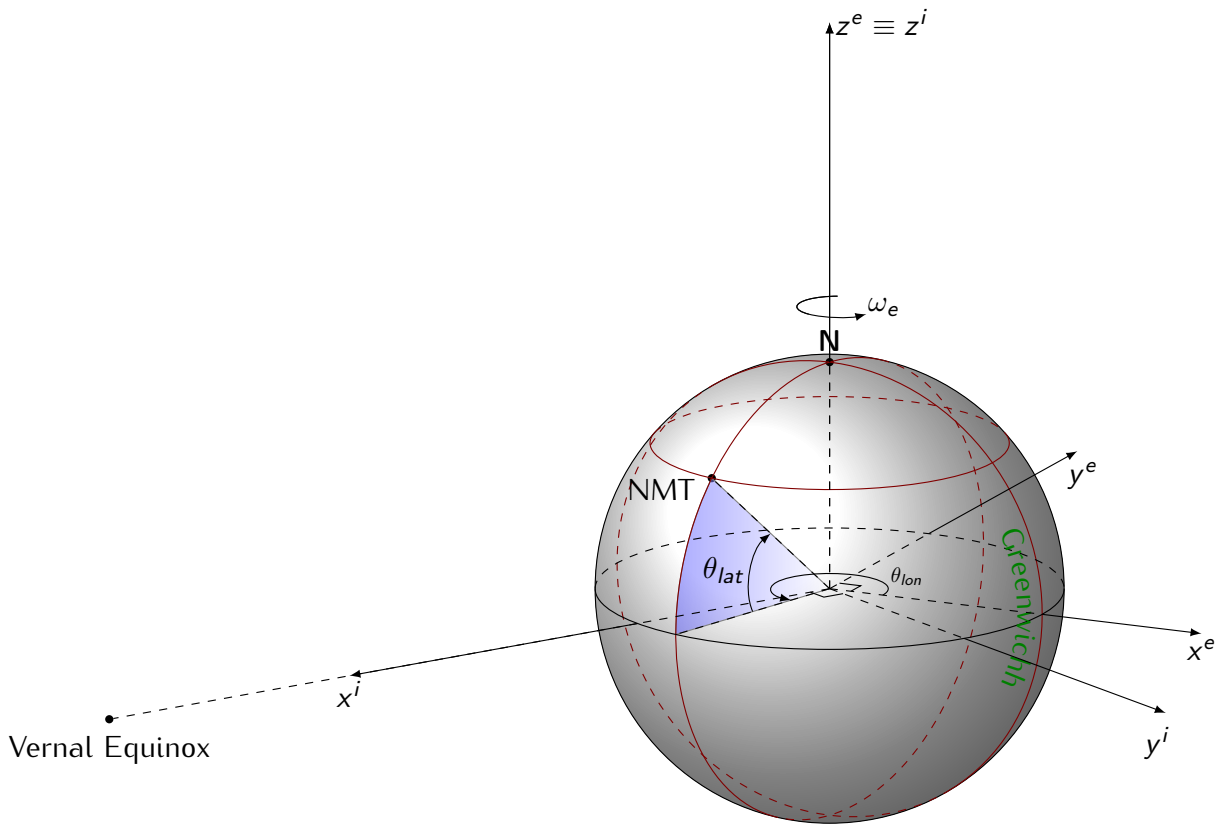
ECEF Coordinate Frame

- z-axis points along the nominal earth axis of rotation
- x-axis points towards zero longitude



ECEF Coordinate Frame

- z-axis points along the nominal earth axis of rotation
- x-axis points towards zero longitude
- y-axis completes a right hand coordinate system



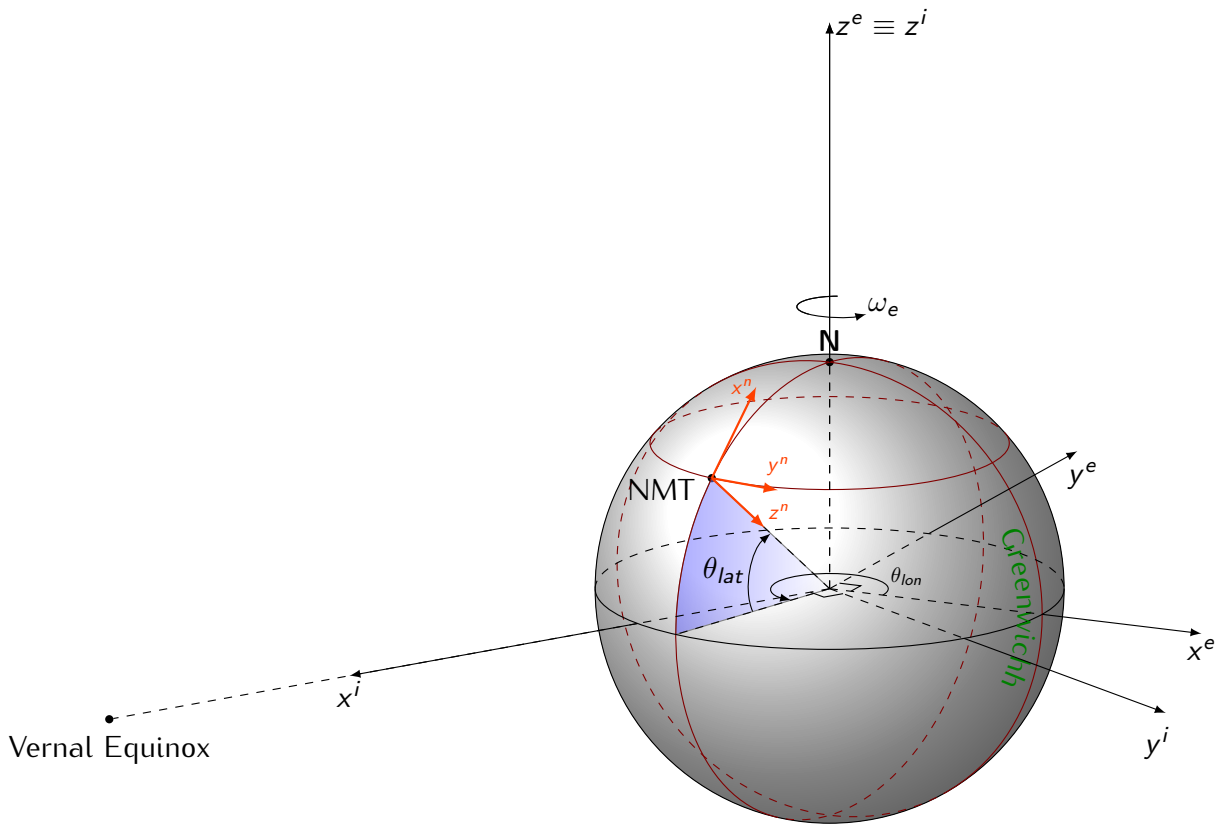
Navigation (Nav) Coordinate Frame

- The Nav coordinate frame is typically **not** fixed with respect to the earth
  - the x/y axis lie in a plane which is locally-level or tangential to the earth's surface
- Nav frame is sometimes call the geodetic, geographic, locally level, or tangential frame
- the Nav frame will be referred to as the n-frame

Nav Coordinate Frame

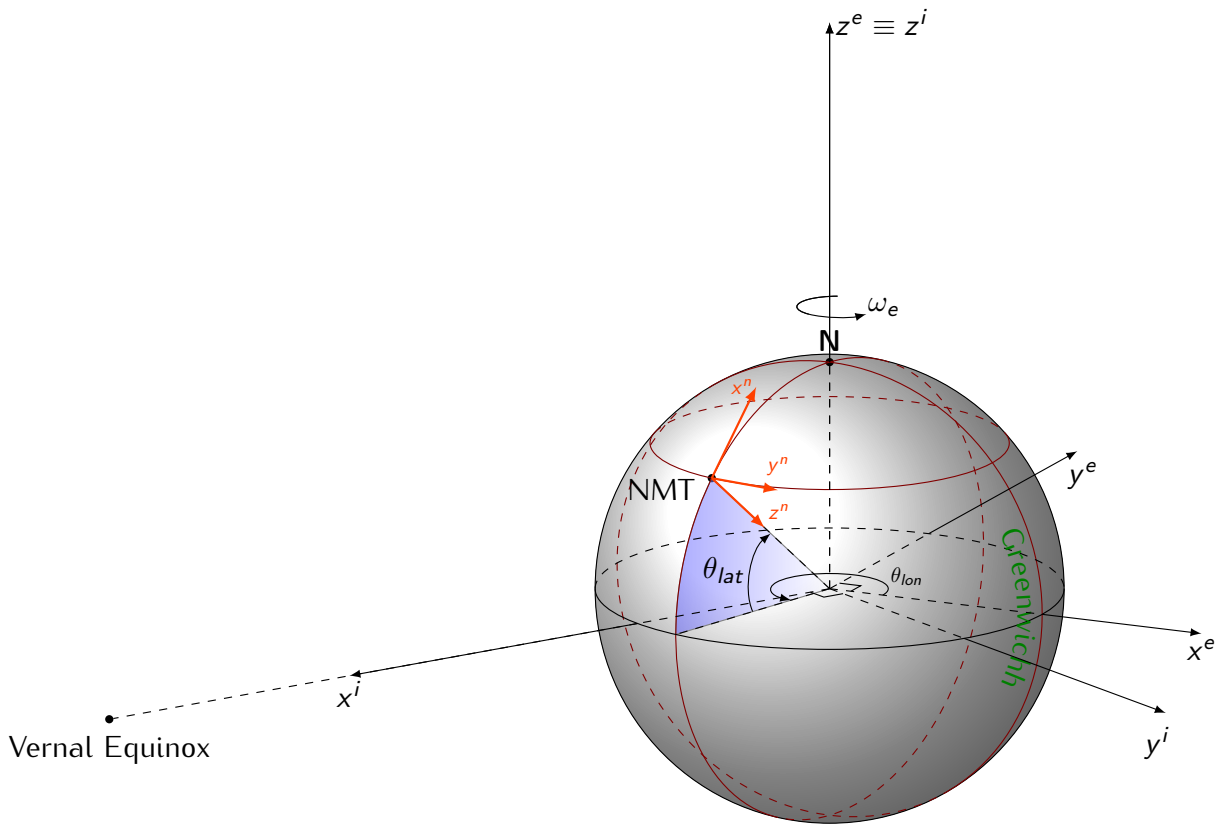
- The origin of the Nav frame is located at the center of mass of the vehicle
- The z-axis points "down" along the normal to the earth's surface
  - Approximately towards the center of the earth
- The x-axis points to the north pole
- The y-axis is simply chosen to conform to a right hand coordinate system
- This configuration is often referred to as the NED frame
  - $x \rightarrow$ north, $y \rightarrow$ East, and $z \rightarrow$ Down

Nav Coordinate Frame



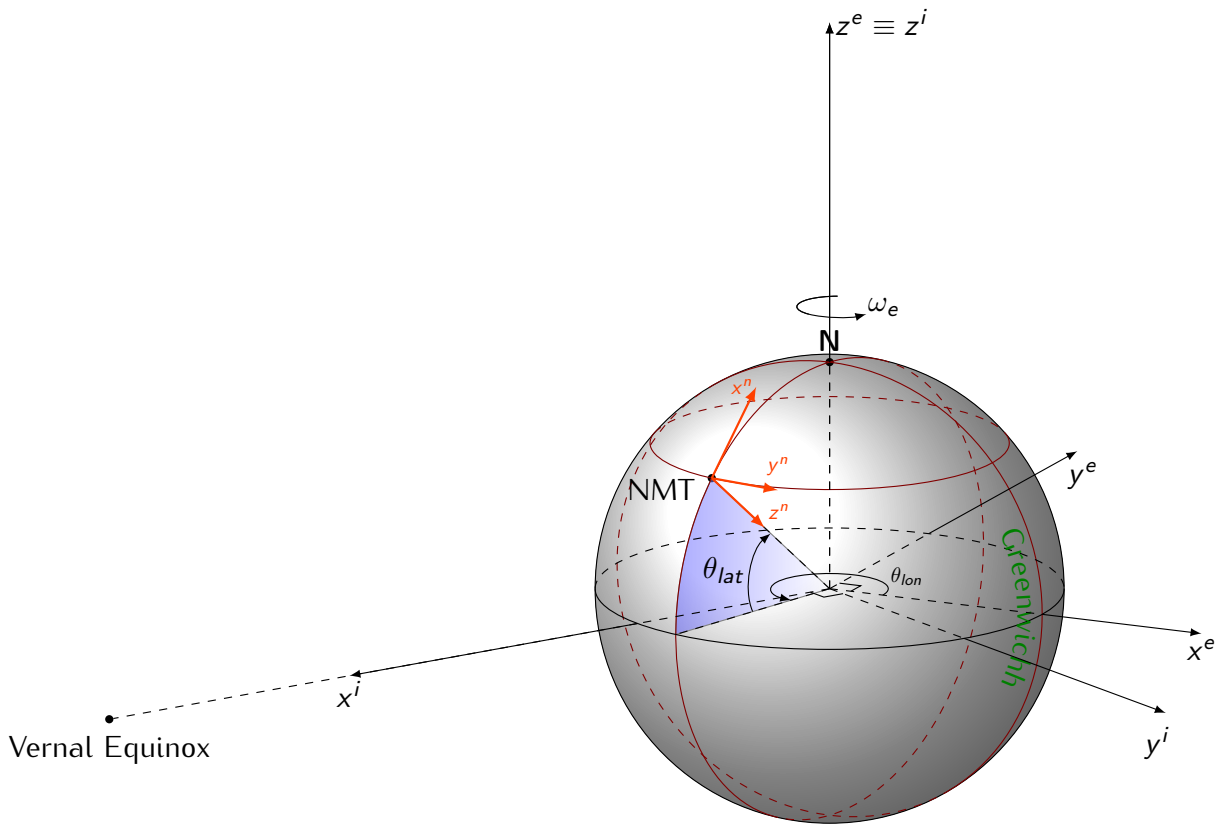
Nav Coordinate Frame

- x-axis points north



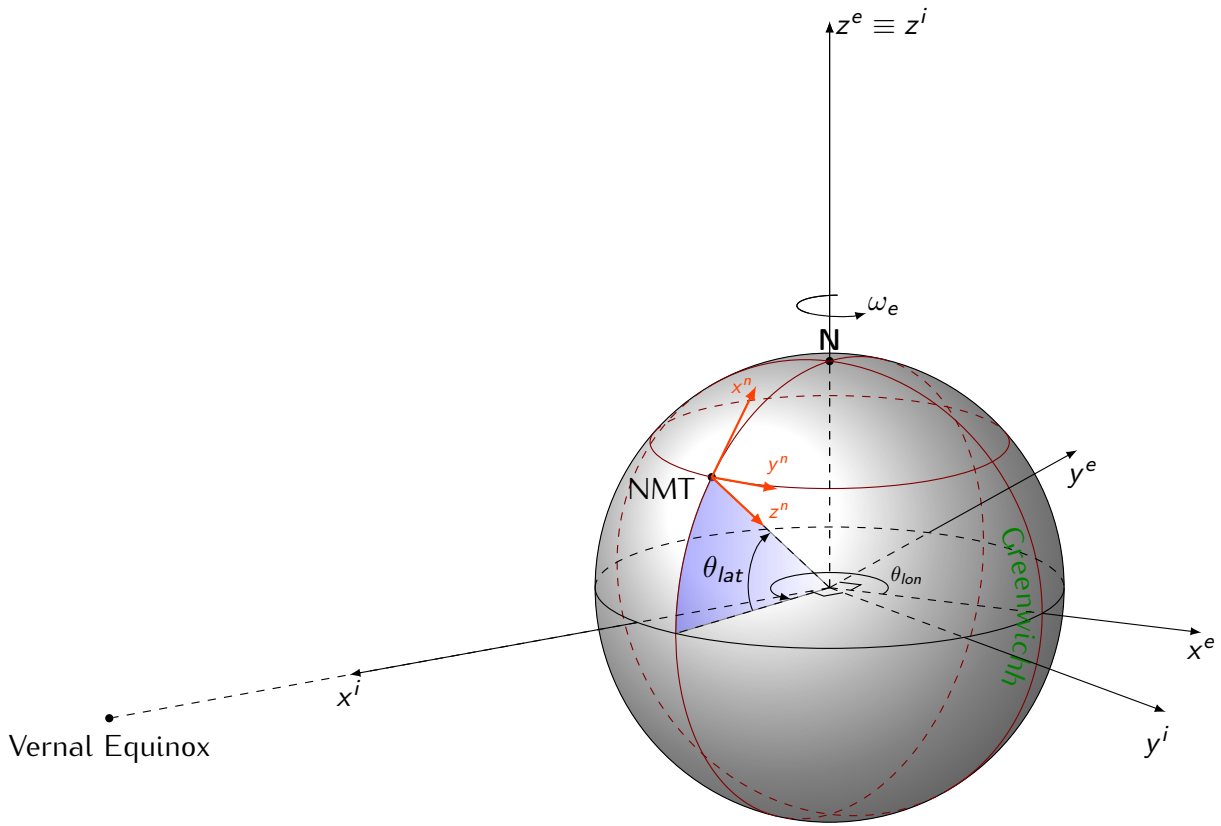
Nav Coordinate Frame

- x-axis points north
- y-axis points east



Nav Coordinate Frame

- x-axis points north
- y-axis points east
- z-axis points down

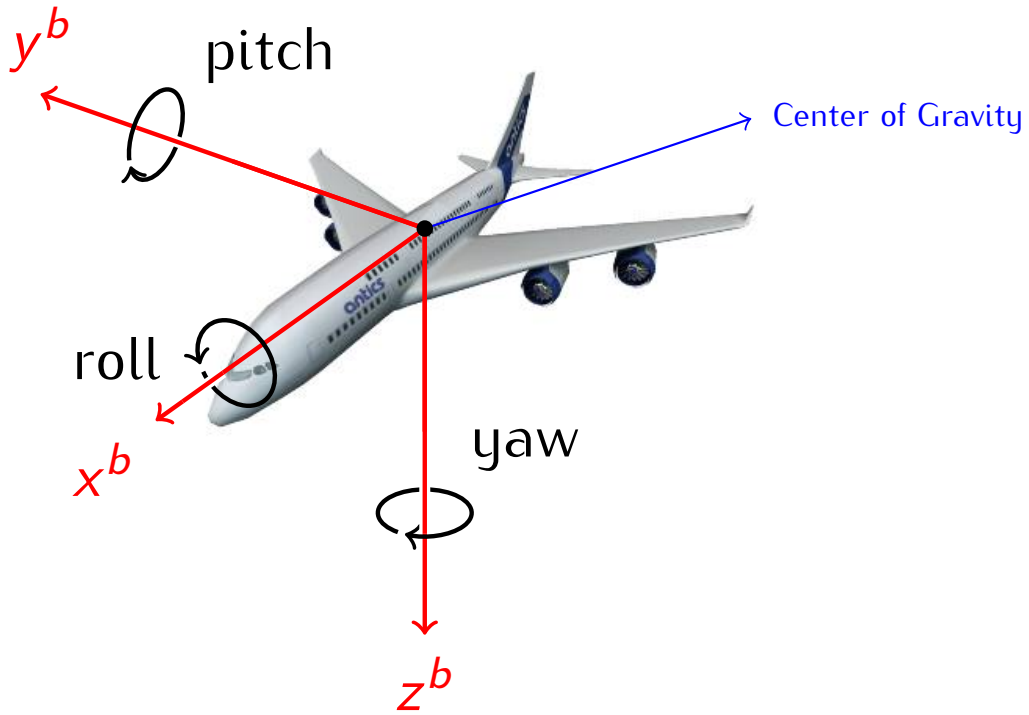


Body Coordinate Frame

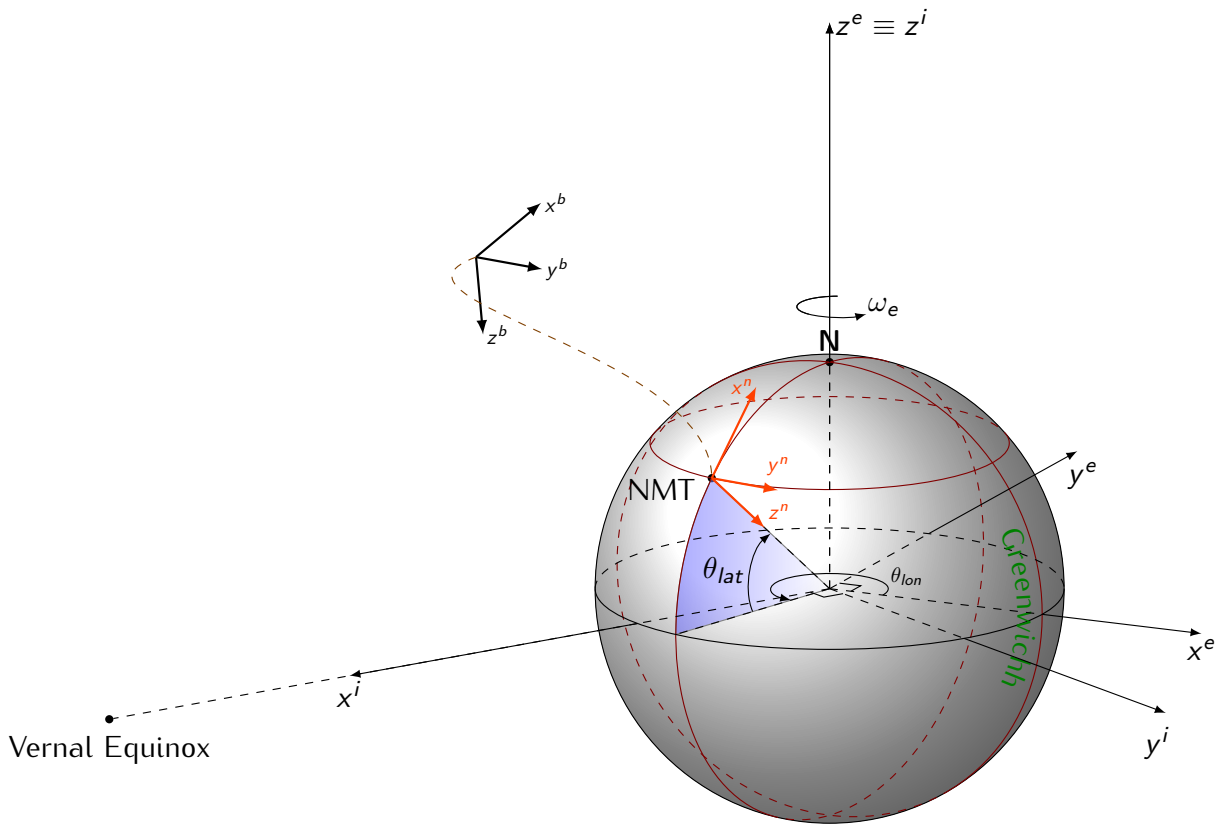
- The origin of the body frame is located at the center of mass of the vehicle (same as the Nav frame)
- The x-axis point "forward" *wrt* the moving vehicle
- The z-axis points loosely "down"
  - Change with the roll/pitch of the vehicle
- The y-axis is simply chosen to conform to a right hand coordinate system

Body Coordinate Frame

- The body coordinate frame is fixed with respect to the vehicle
- The body frame will be referred to as the b-frame

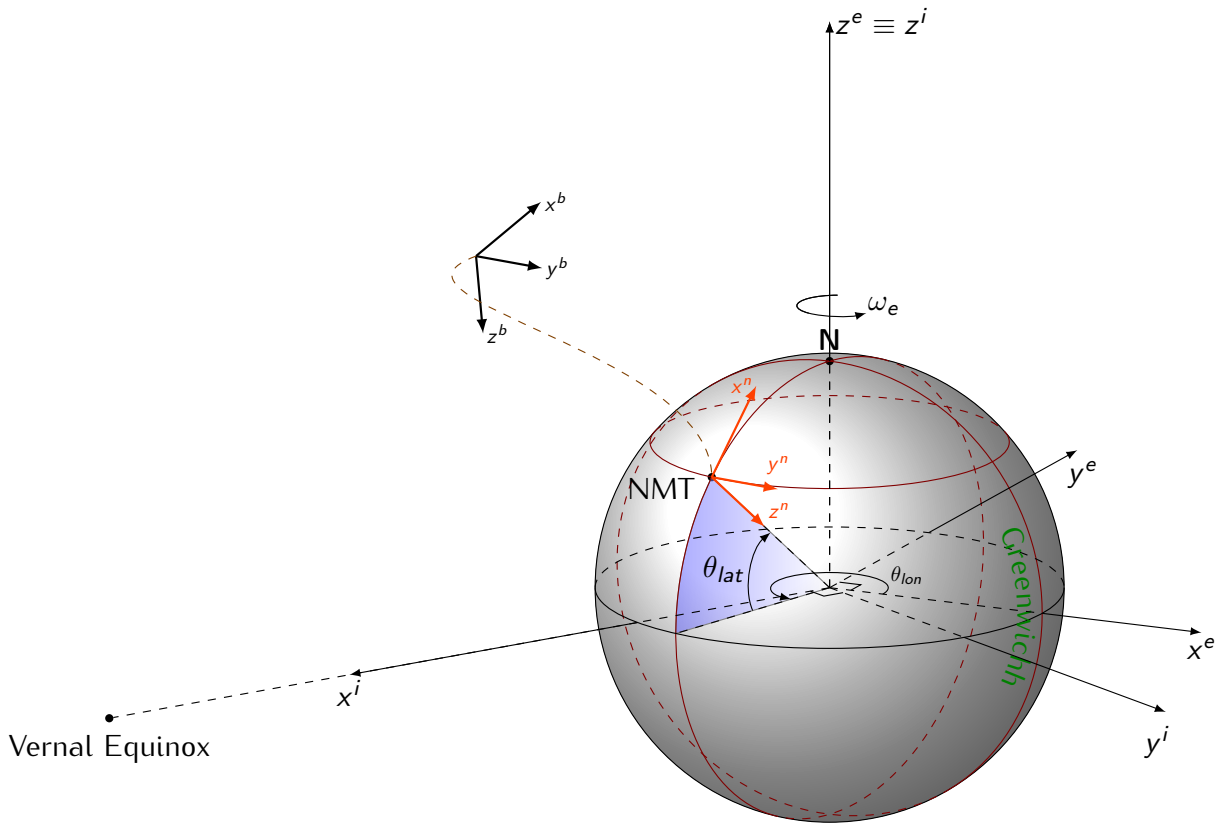


Body Coordinate Frame



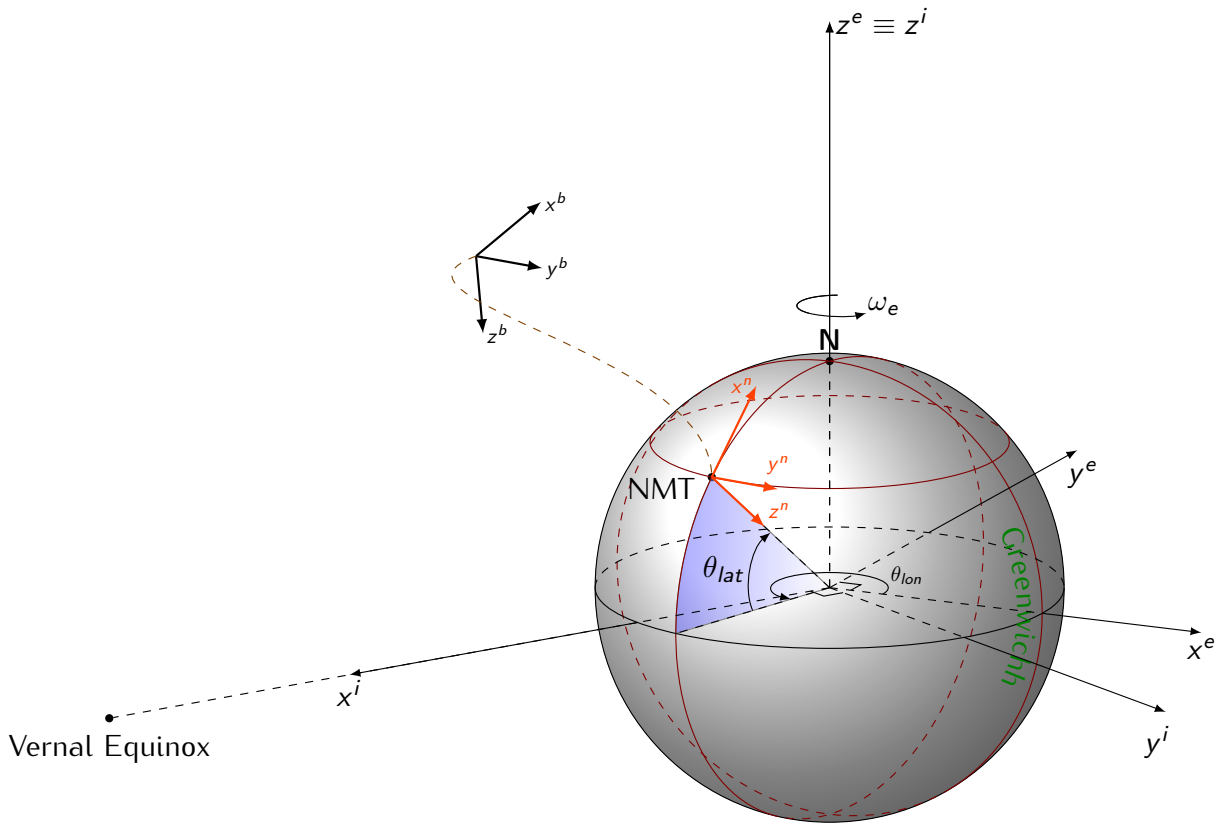
Body Coordinate Frame

- x-axis points forward



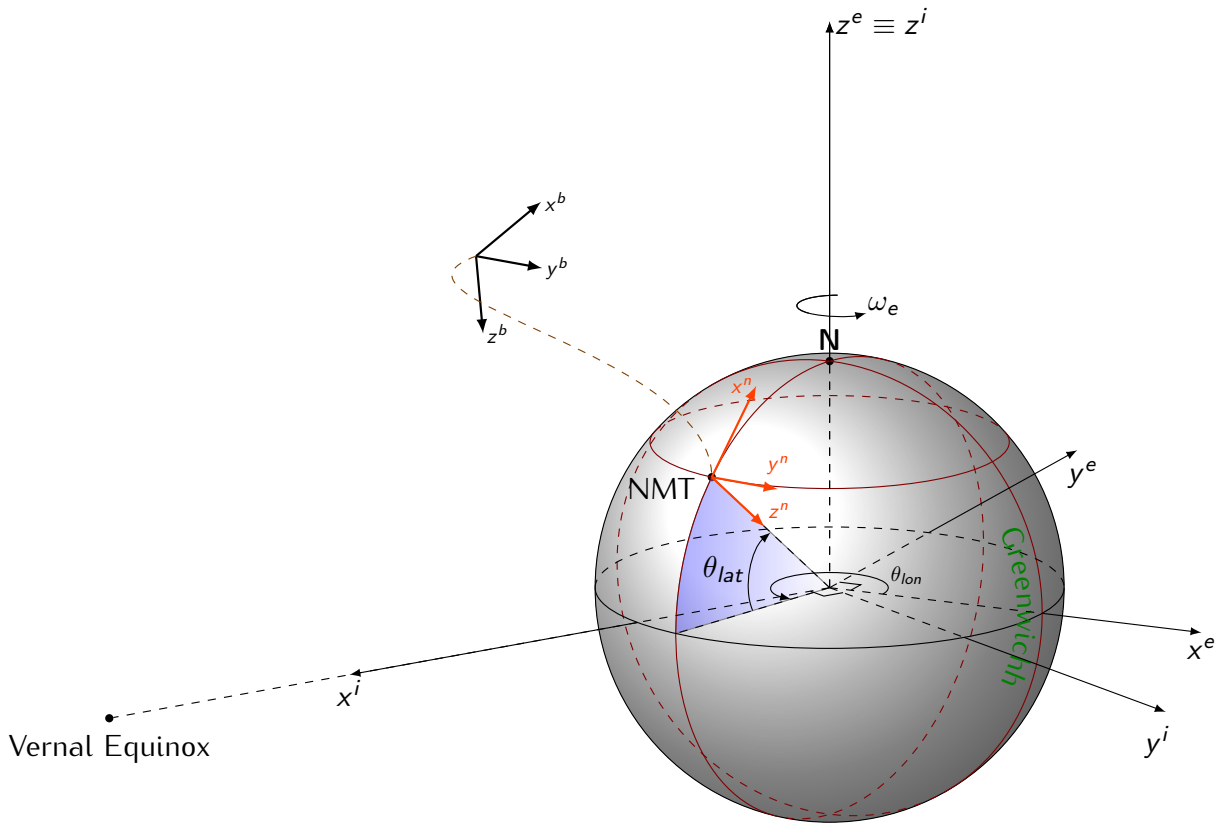
Body Coordinate Frame

- x-axis points forward
- z-axis points "down"



Body Coordinate Frame

- x-axis points forward
- z-axis points "down"
- y-axis points points "right"



Other Coordinate Frames

- Wander Azimuth Frame (alternative to the Nav frame)
  - Does not always point north to avoid numerical stability problems near the poles
- Other locally level frames
  - Tangential Frame
    - Typically, refers to another type of the ECEF frame fixed to the Earth's surface (not moving like the n-frame)
  - Computer Frame
    - Virtual coordinate frame that represents where we think that we are