EE 570: Location and Navigation

Navigation Mathematics: Kinematics (Translation)

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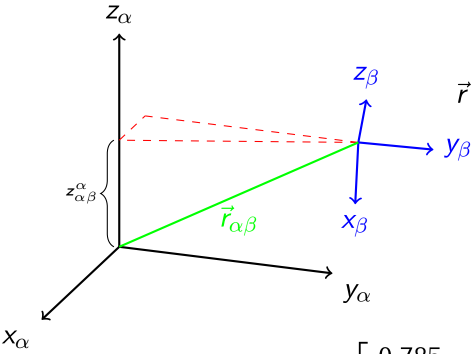
- $\vec{r}_{\alpha\beta}$ is the vector from the origin of $\{\alpha\}$ to the origin of $\{\beta\}$.

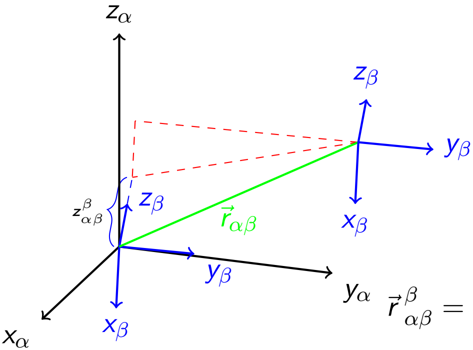


$$\vec{r}^{\,\alpha}_{\alpha\beta} = \begin{bmatrix} x^\alpha_{\alpha\beta} \\ y^\alpha_{\alpha\beta} \\ z^\alpha_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} -1.00 \\ 3.00 \\ 1.50 \end{bmatrix}$$

$$C^\alpha_\beta = \begin{bmatrix} 0.785 & -0.366 & 0.500 \\ 0.242 & 0.924 & 0.296 \\ -0.571 & -0.111 & 0.813 \end{bmatrix}$$

Translation

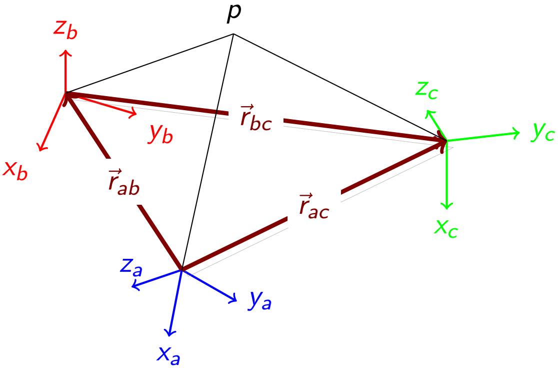
- Resolve, i.e., coordinatize, $\vec{r}_{\alpha\beta}$ *wrt* frame $\{\beta\}$.



$$\vec{r}^{\,\beta}_{\alpha\beta} = \begin{bmatrix} x^\beta_{\alpha\beta} \\ y^\beta_{\alpha\beta} \\ z^\beta_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} -0.914 \\ 2.97 \\ 1.61 \end{bmatrix} = C^\beta_\alpha \vec{r}^{\,\alpha}_{\alpha\beta}$$

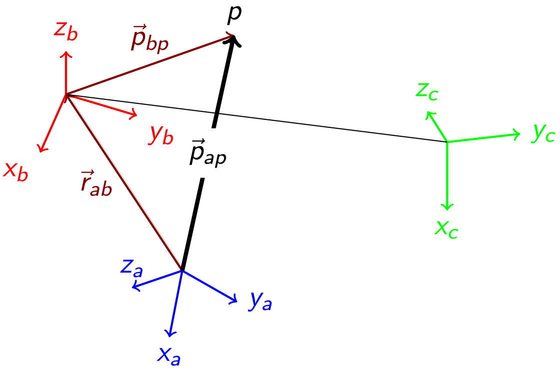
The two vectors have the same length

- Consider the three coordinate systems $\{a\}$, $\{b\}$, and $\{c\}$
  - Origin from $\{a\}$ to $\{b\}$ and vice-versa, i.e., $\vec{r}_{ab} = -\vec{r}_{ba}$
  - Knowing the relationship between frames $\{a\}$, $\{b\}$, and $\{c\}$, i.e., $\vec{r}_{ab}$, $\vec{r}_{bc}$, $\vec{r}_{ac}$, C_b^a, C_c^b, and C_c^a
  - Determine the location of the point p
  - Describe in what frame,? i.e., $\vec{p}^{\,a}$ or $\vec{p}^{\,b}$ or $\vec{p}^{\,c}$.



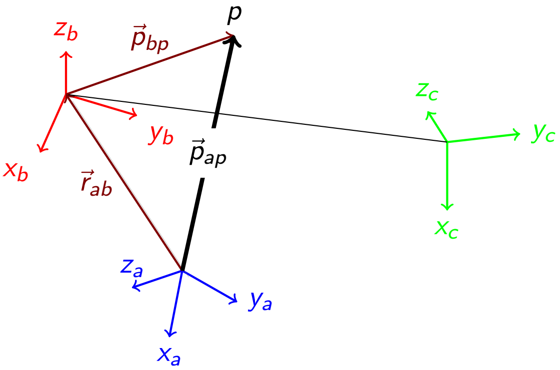
- Determine the location of the point p
  - Origin from $\{a\}$ to point p

- $\vec{p}_{ap} = \vec{r}_{ab} + \vec{p}_{bp}$
- In what frame?
- $\vec{p}^{\,a}_{ap} = \vec{r}^{\,a}_{ab} + \vec{p}^{\,a}_{bp}$
- $\vec{p}^{\,a} \equiv \vec{p}^{\,a}_{bp}$



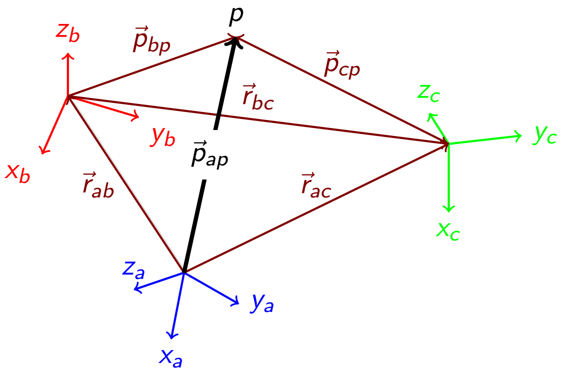
- Determine the location of the point p
    - Origin from $\{b\}$ to point p

- $\vec{p}_{ap} = \vec{r}_{ab} + \vec{p}_{bp}$
- In what frame?
- $\vec{p}^{\,b}_{ap} = \vec{r}^{\,b}_{ab} + \vec{p}^{\,b}_{bp}$
- $\vec{p}^{\,b} \equiv \vec{p}^{\,b}_{bp} = \vec{p}^{\,b}_{ap} - \vec{r}^{\,b}_{ab}$
- $\vec{p}^{\,b}_{bp} = C^b_a \vec{p}^{\,a}_{ap}$



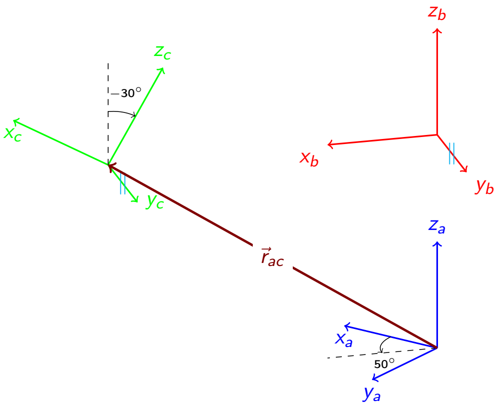
- Determine the location of the point p
  - Origin from $\{c\}$ to point p

- $\vec{p}_{ap} = \vec{r}_{ac} + \vec{p}_{cp}$
- In what frame?
- $\vec{p}^{\,c}_{ap} = \vec{r}^{\,c}_{ab} + \vec{p}^{\,c}_{bp}$
- $\vec{p}^{\,c} \equiv \vec{p}^{\,c}_{cp} = \vec{p}^{\,c}_{ap} - \vec{r}^{\,c}_{ac}$
- $\vec{p}^{\,a}_{cp} = C^a_c \vec{p}^{\,c}_{cp}$
- $\vec{p}^{\,b}_{cp} = C^b_c \vec{p}^{\,c}_{cp}$

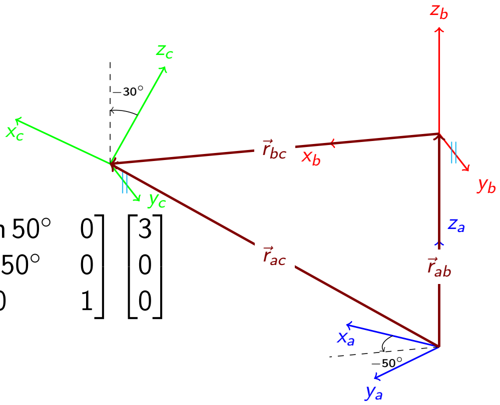


Example

- $C_b^a = R_{(\vec{z},50^\circ)}$
- $C_c^b = R_{(\vec{y},-30^\circ)}$
- $C_c^a = C_b^a C_c^b$
- $\vec{r}\,^a_{ab} = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}^T$
- $\vec{r}\,^b_{bc} = \begin{bmatrix} 3 & 0 & 0 \end{bmatrix}^T$
- find $\vec{r}\,^a_{ac}$
- find $\vec{r}\,^c_{ca}$



$$
\begin{aligned}
\vec{r}^{\,a}_{ac} &= \vec{r}^{\,a}_{ab} + \vec{r}^{\,a}_{bc} \\
&= \vec{r}^{\,a}_{ab} + C^a_b \vec{r}^{\,b}_{bc} \\
&= \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + R_{(\vec{z},50^\circ)} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} \cos 50^\circ & -\sin 50^\circ & 0 \\ \sin 50^\circ & \cos 50^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 1.93 \\ 2.30 \\ 2.00 \end{bmatrix}
\end{aligned}
$$



$$
\begin{aligned}
\vec{r}^{\,c}_{ca} &= -\vec{r}^{\,c}_{ac} \\
&= -C^c_a \vec{r}^{\,a}_{ac} \\
&= -\left(C^a_c\right)^T \vec{r}^{\,a}_{ac} \\
&= -\left(R_{(\vec{z},50^\circ)}, R_{(\vec{y},-30^\circ)}\right)^T \begin{bmatrix} 1.93 \\ 2.30 \\ 2.00 \end{bmatrix} \\
&= \begin{bmatrix} -3.59 \\ 0 \\ -0.232 \end{bmatrix}
\end{aligned}
$$