Lecture

Navigation Equations: ECI Mechanization

EE 570: Location and Navigation

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ECI Mechanization

- Determine the position, velocity and attitude of the **body** frame *wrt* the **inertial** frame.
  - **Position** — Vector from the origin of the inertial frame to the origin of the body frame resolved in the inertial frame: $\vec{r}^{\,i}_{ib}$
  - **Velocity** — Velocity of the body frame *wrt* the inertial frame resolved in the inertial frame: $\vec{v}^{\,i}_{ib}$
  - **Attitude** — Orientation of the body frame *wrt* the inertial frame: C^i_b

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Attitude — Method A

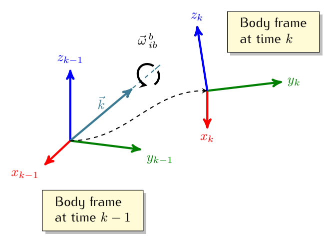
- Body orientation frame at time "k" *wrt* time "$k-1$"
  - $\Delta t = t_k - t_{k-1}$

$$\dot{C}^i_b = C^i_b \Omega^b_{ib}$$

$$= \lim_{\Delta \to 0} \left(\frac{C^i_b(k) - C^i_b(k-1)}{\Delta t} \right) = C^i_b(k-1)\Omega^b_{ib}$$

$$C^i_b(+) - C^i_b(-) \approx C^i_b(-)\Omega^b_{ib}\Delta t$$

$$C^i_b(+) \approx C^i_b(-)\left(\mathcal{I} + \Omega^b_{ib}\Delta t\right)$$



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Attitude — Method B

- Body orientation frame at time "k" *wrt* time "$k-1$"
  - $\Delta t = t_k - t_{k-1}$

$$\boxed{\vec{\omega}^{\,b}_{ib}\Delta t = \vec{k}\Delta\theta}$$

$$C^i_{b(k)} = C^i_{b(k-1)} C^{b(k-1)}_{b(k)}$$

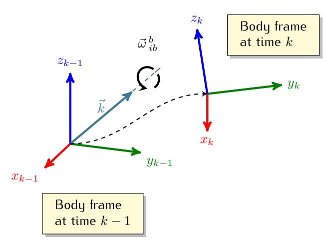
$$C^{b(k-1)}_{b(k)} = e^{\Omega^b_{ib}\Delta t} = e^{\mathfrak{K}\Delta\theta}$$

$$= \mathcal{I} + \mathfrak{K}\Delta\theta + \frac{\mathfrak{K}^2\Delta\theta^2}{2!} + \frac{\mathfrak{K}^3\Delta\theta^3}{3!} + \dots$$

$$= \mathcal{I} + \sin(\Delta\theta)\mathfrak{K} + \left[1 - \cos(\Delta\theta)\right]\mathfrak{K}^2$$

$$C^i_b(+) = C^i_b(-)C^{b(k-1)}_{b(k)}$$

$$\approx C^i_b(-)\left(\mathcal{I} + \Omega^b_{ib}\Delta t\right)$$



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Attitude — Method C

- Body orientation frame at time "k" *wrt* time "$k-1$"
  - $\Delta t = t_k - t_{k-1}$

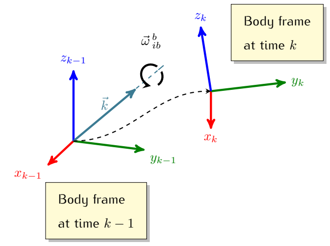
$\vec{\omega}^{\,b}_{ib}\Delta t = \vec{k}\Delta\theta$

$$\bar{q}^{\,i}_{\,b(k)} = \bar{q}^{\,i}_{\,b(k-1)} \otimes \bar{q}^{\,b(k-1)}_{\,b(k)}$$

$$\bar{q}^{\,b(k-1)}_{\,b(k)} = \begin{bmatrix} \cos(\frac{\Delta\theta}{2}) \\ \vec{k}\sin(\frac{\Delta\theta}{2}) \end{bmatrix}$$

$$\bar{q}^{\,i}_{\,b}(+) = \bar{q}^{\,i}_{\,b}(-) \otimes \bar{q}^{\,b(k-1)}_{\,b(k)}$$

Need to periodically renormalize $\bar{q}$



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Attitude Update— Summary

- High fidelity

$$C^i_b(+) = C^i_b(-)\left[\mathcal{I} + \sin(\Delta\theta)\mathfrak{K} + [1-\cos(\Delta\theta)]\,\mathfrak{K}^2\right] \tag{1}$$

or

$$\bar{q}^{\,i}_{\,b}(+) = \bar{q}^{\,i}_{\,b}(-) \otimes \begin{bmatrix} \cos(\frac{\Delta\theta}{2}) \\ \vec{k}\sin(\frac{\Delta\theta}{2}) \end{bmatrix} \tag{2}$$

- Low fidelity

$$C^i_b(+) \approx C^i_b(-)\left(\mathcal{I} + \Omega^b_{ib}\Delta t\right) \tag{3}$$

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Steps 2–4

2. Specific force transformation
   - Simply coordinatize the specific force

$$\vec{f}^{\,i}_{ib} = C^i_b(+)\vec{f}^{\,b}_{ib} \tag{4}$$

3. Velocity update
   - Assuming that we are in space (i.e., no centrifugal component)

$$\vec{a}^{\,i}_{ib} = \vec{f}^{\,i}_{ib} + \vec{\gamma}^{\,i}_{ib} \tag{5}$$

   - Thus, by simple numerical integration

$$\vec{v}^{\,i}_{ib}(+) = \vec{v}^{\,i}_{ib}(-) + \vec{a}^{\,i}_{ib}\Delta t \tag{6}$$

4. Position update
   - by simple numerical integration

$$\vec{r}^{\,i}_{ib}(+) = \vec{r}^{\,i}_{ib}(-) + \vec{v}^{\,i}_{ib}(-)\Delta t + \vec{a}^{\,i}_{ib}\frac{\Delta t^2}{2} \tag{7}$$

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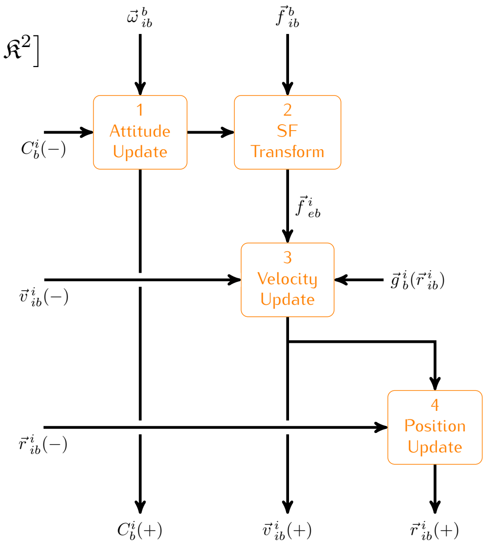
ECI Mechanization Summary



$$C^i_b(+) = C^i_b(-)\left[\mathcal{I} + \sin(\Delta\theta)\mathfrak{K} + [1-\cos(\Delta\theta)]\,\mathfrak{K}^2\right]$$

or

$$C^i_b(+) \approx C^i_b(-)\left(\mathcal{I} + \Omega^b_{ib}\Delta t\right)$$

or

$$\bar{q}^{\,i}_b(+) = \bar{q}^{\,i}_b(-) \otimes \begin{bmatrix} \cos(\frac{\Delta\theta}{2}) \\ \vec{k}\sin(\frac{\Delta\theta}{2}) \end{bmatrix}$$

and

$$\vec{f}^{\,i}_{ib} = C^i_b(+)\vec{f}^{\,b}_{ib}$$

$$\vec{a}^{\,i}_{ib} = \vec{f}^{\,i}_{ib} + \vec{\gamma}^{\,i}_{ib}$$

$$\vec{v}^{\,i}_{ib}(+) = \vec{v}^{\,i}_{ib}(-) + \vec{a}^{\,i}_{ib}\Delta t$$

$$\vec{r}^{\,i}_{ib}(+) = \vec{r}^{\,i}_{ib}(-) + \vec{v}^{\,i}_{ib}(-)\Delta t + \vec{a}^{\,i}_{ib}\frac{\Delta t^2}{2}$$

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ECI Mechanization — Continuous Case

- In continuous time notation
  - Attitude: $\dot{C}^i_b = C^i_b\Omega^b_{ib}$ or $\dot{\bar{q}}^i_b = \frac{1}{2}[\breve{\omega}^b_{ib}\circledast]\bar{q}^{\,i}_b(t)$
  - Velocity: $\dot{\vec{v}}^{\,i}_{ib} = C^i_b\vec{f}^{\,b}_{ib} + \vec{\gamma}^{\,i}_{ib}$
  - Position: $\dot{\vec{r}}^{\,i}_{ib} = \vec{v}^{\,i}_{ib}$
- In State-space notation

$$\begin{bmatrix} \dot{\vec{r}}^{\,i}_{ib} \\ \dot{\vec{v}}^{\,i}_{ib} \\ \dot{C}^i_b \end{bmatrix} = \begin{bmatrix} \vec{v}^{\,i}_{ib} \\ C^i_b\vec{f}^{\,b}_{ib} + \vec{\gamma}^{\,i}_{ib} \\ C^i_b\Omega^b_{ib} \end{bmatrix} \tag{8}$$

or

$$\begin{bmatrix} \dot{\vec{r}}^{\,i}_{ib} \\ \dot{\vec{v}}^{\,i}_{ib} \\ \dot{\bar{q}}^i_b \end{bmatrix} = \begin{bmatrix} \vec{v}^{\,i}_{ib} \\ C^i_b\vec{f}^{\,b}_{ib} + \vec{\gamma}^{\,i}_{ib} \\ \frac{1}{2}[\breve{\omega}^b_{ib}\circledast]\bar{q}^{\,i}_b(t) \end{bmatrix} \tag{9}$$

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Appendix

$$[\bar{q}\otimes] = \begin{bmatrix} q_s & -q_x & -q_y & -q_z \\ q_x & q_s & -q_z & q_y \\ q_y & q_z & q_s & -q_x \\ q_z & -q_y & q_x & q_s \end{bmatrix}$$

$$[\bar{q}\circledast] = \begin{bmatrix} q_s & -q_x & -q_y & -q_z \\ q_x & q_s & q_z & -q_y \\ q_y & -q_z & q_s & q_x \\ q_z & q_y & -q_x & q_s \end{bmatrix}$$

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