# Lecture

## Gyro and Accel Noise Characteristics

### EE 570: Location and Navigation

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## 1 Gyro Noise Characteristics

### Gyro Constant Bias ($^\circ/h$)

A constant in the output of a gyro in the absence of rotation, in $^\circ/h$.

**Error Growth**
Linearly growing error in the angle domain of $\epsilon t$.

**Model**
Random constant.

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### Gyro Integrated White Noise

Assuming the rectangular rule is used for integration, a sampling period of $T_s$ and a time span of $nT_s$.

$$\int_0^t \epsilon(\tau)d\tau = T_s \sum_{i=1}^{n} \epsilon(t_i) \tag{1}$$

since $\mathbb{E}[\epsilon(t_i)] = 0$ and $Cov(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $Var[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E}\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s n \mathbb{E}[\epsilon(t_i)] = 0, \forall i \tag{2}$$

$$Var\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s^2 n Var[\epsilon(t_i)] = T_s t \sigma^2, \forall i \tag{3}$$

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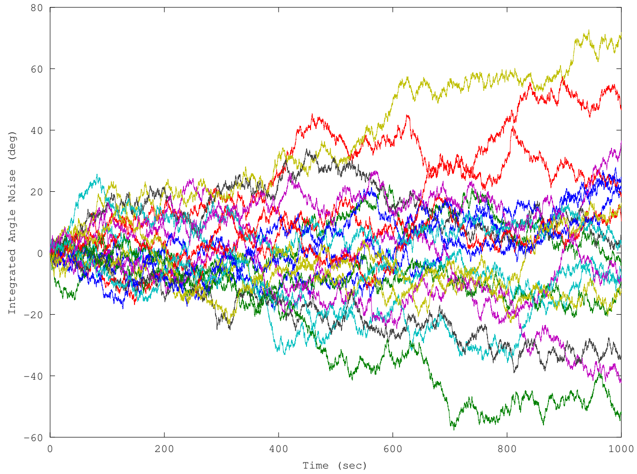
### Gyro Integrated White Noise

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### Angle Random Walk ($^\circ/\sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma\sqrt{T_s t} \tag{4}$$

We define $ARW$ as

$$ARW = \sigma_\theta(1) \qquad (^\circ/\sqrt{h}) \tag{5}$$

In terms of PSD

$$ARW(^\circ/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((^\circ/h)^2/Hz)} \tag{6}$$

**Error Growth**
*ARW times root of the time in hours.*

**Model**
White noise. .5

### Gyro Bias Instability ($^\circ/h$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

**Error Growth**
*Variance grows over time.*

**Model**
First order Gauss-Markov. .6

## 2 Accel Noise Characteristics

### Accel Constant Bias ($\mu g$)

A constant deviation in the accelerometer from the true value, in $m/s^2$.

**Error growth**
Double integrating a constant bias error of $\epsilon$ results in a quadratically growing error in position of $\epsilon t^2/2$.

**Model**
Random constant. .7

### Velocity Random Walk $(m/s/\sqrt{h})$

Integrating accelerometer output containing white noise results in velocity random walk (VRW) $(m/s/\sqrt{h})$. Similar to development of ARW, if we double integrate white noise we get

$$\iint_0^t \epsilon(\tau)d\tau d\tau = T^2_{s,sensor} \sum_{i=1}^{n} \sum_{j=1}^{i} \epsilon(t_j) \tag{7}$$

**Error Growth**
Computing the variance results in

$$\sigma_p \approx \sigma t^{(3/2)} \sqrt{\frac{T_s}{3}} \tag{8}$$

**Model**
White noise. .8

### Accel Bias Stability $(\mu g)$

**Error growth**
Grows as $t^{5/2}$.

**Model**
First order Gauss-Markov. .9

## 3 Allan Variance

### Allan Variance Introduction

It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966. .10

### Allan Variance Computation

1. Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \dots, M \leq N/2$.
2. For every $n$ generate the sequence

$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \cdots + x_{nj+n-1}}{n}, \quad j = 0, 1, \dots, \left[\frac{N}{n}\right] - 1 \tag{9}$$

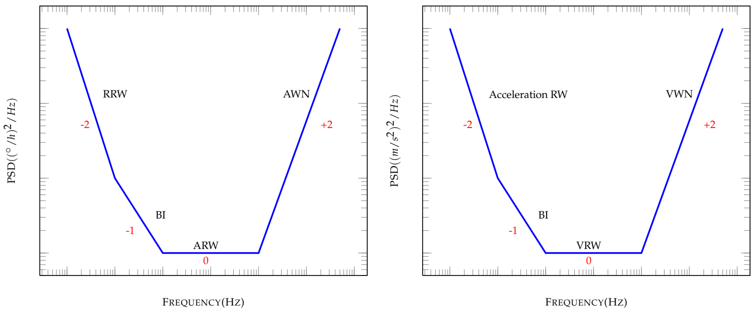
3. Plot log-log of the Allan deviation which is square root of

$$\sigma^2_{Allan}(nT_s) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_j - y_{j-1})^2 \tag{10}$$

versus averaging time $\tau = nT_s$ .11

# 4 Using PSD and Allan Variance

## One-sided PSD - Typical Slopes

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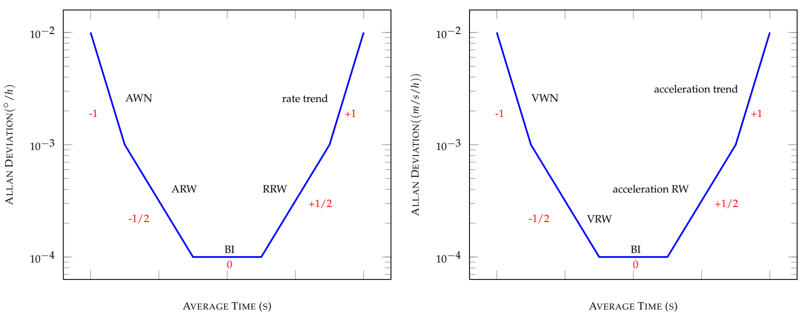
## Allan Deviation - Typical Slopes

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## Noise Parameters

| **Noise Type** | AV $\sigma^2(\tau)$ | PSD (2-sided) |
|---|---|---|
| Quantization Noise | $3\frac{\alpha^2}{\tau^2}$ | $(2\pi f)^2\alpha^2 T_s$ |
| Angle/Velocity Random Walk | $\frac{\alpha^2}{\tau}$ | $\alpha^2$ |
| Flicker Noise | $\frac{2\alpha^2\ln(2)}{\pi}$ | $\frac{\alpha^2}{2\pi f}$ |
| Angular Rate/Accel Random Walk | $\frac{\alpha^2\tau}{3}$ | $\frac{\alpha^2}{(2\pi f)^2}$ |
| Ramp Noise | $\frac{\alpha^2\tau^2}{2}$ | $\frac{\alpha^2}{(2\pi f)^3}$ |

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