1. Describe each of the following rotations using roll-pitch-yaw angles ($\phi,\ \theta,\ \psi$), angle-axis $\left(\theta,\ \vec{k}\right)$ and quaternion $\bar{q}$.
   (a) no rotation
   (b) rotation about x by $90°$
   (c) rotation about y by $180°$
   (d) rotation about z by $-90°$

2. Consider the quaternion $\bar{q}_{x,90°}$ that describes a rotation about the x-axis by $90°$.
   (a) Show the opposite rotation $\bar{q}_{x,-90°}$ is its inverse (conjugate) using the definition.
   (b) Compose $\bar{q}_{x,90°}$ with $\bar{q}_{x,-90°}$ via multiplication ($\otimes$) on the left and right. Do both yield the identity quaternion?
   (c) Given $\bar{q}^0_1 = \bar{q}_{x,-90°}$ describes the orientation of frame $\{1\}$ relative to frame $\{0\}$, recoordinatize the vector $\vec{v}^1 = [1,\ 1,\ 1]^T$ in frame $\{0\}$, i.e., use the quaternion to find $\vec{v}^0$.

3. How many multiplies and additions are needed for each of the following computations?
   (a) composition of rotations via rotation matrices, $C^2_1 C^1_0$
   (b) composition of rotations via quaternions, $\bar{q}^2_1 \otimes \bar{q}^1_0$
   (c) recoordinatization of a vector via rotation matrix, $C^2_1 \vec{r}^1$
   (d) recoordinatization of a vector via quaternion, $\bar{q}^2_1 \otimes \check{r}^1 \otimes (\bar{q}^2_1)^{-1}$

4. Consider the time-varying rotation matrix $C^a_b = R_{z,\theta(t)}$ that describes the orientation of frame {b} as it rotates about frame {a}'s $z-$axis by angle $\theta(t)$. Determine the angular velocity as a skew-symmetric matrix (via $\Omega^a_{ab} = \dot{C}^a_b [C^a_b]^T$) and vector $\vec{\omega}^a_{ab}$. Do these answers make sense in light of the type of rotation between the frames?

5. Consider the time-varying coordinate transformation matrix C_b^n given below that describes the orientation of the body frame as it rotates with respect to the navigation frame.

$$C_b^n = \begin{bmatrix} \cos(t) & \sin(t)\sin(t^2) & \sin(t)\cos(t^2) \\ 0 & \cos(t^2) & -\sin(t^2) \\ -\sin(t) & \cos(t)\sin(t^2) & \cos(t)\cos(t^2) \end{bmatrix}$$

   (a) Determine the analytic form of the time-derivative of C_b^n (i.e., $\dot{C}_b^n = \frac{dC_b^n}{dt}$) via a term-by-term differentiation.

   (b) Develop MATLAB functions that accept "t" (i.e., time) as a numerical input and return C_b^n and $\dot{C}_b^n$, respectively, as numerical outputs.

   (c) Using the C_b^n and $\dot{C}_b^n$ functions from above, compute the angular velocity vector $\vec{\omega}_{nb}^n$ at time $t = 0$ sec (**Hint**: you might want to compute Ω_{nb}^n first).

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^n$) has the instantaneous rotation occurred?

   (d) Using the C_b^n and $\dot{C}_b^n$ functions from above, compute the angular velocity vector $\vec{\omega}_{nb}^n$ at time $t = 0.5$ sec.

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^n$) has the instantaneous rotation occurred?

   (e) Using C_b^n and $\dot{C}_b^n$ functions from above, compute the angular velocity vector $\vec{\omega}_{nb}^n$ at time $t = 1$ sec.

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^n$) has the instantaneous rotation occurred?

   (f) In practice, direct measurement of the angular velocity vector $\vec{\omega}_{nb}^n$ can prove challenging, so a finite-difference approach may be taken given two sequential orientations represented by $C_b^n(t)$ and $C_b^n(t+\Delta t)$ a small time Δt apart. Consider the approximate value of the angular velocity vector $\vec{\omega}_{nb}^n$ derived by using the finite difference

$$\dot{C}_b^n(t) \approx \frac{C_b^n(t+\Delta t) - C_b^n(t)}{\Delta t}$$

   at times $t = 0, 0.5$, and 1 sec. Compare the "analytic" values for $\dot{\theta}$ and $\vec{k}_{nb}^n$ (found in parts b, c and d) with your approximations from the finite difference using $\Delta t = 0.1$ sec. How large are the errors?

   (g) How small does the sampling time (i.e., Δt) need to be to get a "good" (better than 99.9%) approximation of the angular speed (i.e., $\dot{\theta}$)?

6. Consider the three coordinate frames $\{\alpha\}$, $\{\beta\}$, and $\{\gamma\}$ shown in the diagram below. Following the notation introduced in the class, find the following Cartesian position vectors (denoted by $\vec{r}$).

   (a) $\vec{r}^{\gamma}_{\gamma\alpha}$

   (b) $\vec{r}^{\gamma}_{\gamma\beta}$

   (c) $\vec{r}^{\beta}_{\beta\alpha}$ by appropriately adding vectors in parts (a) and (b)