

# EE 565: Position, Navigation and Timing

## Error Modeling

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Given a state space system

$$\dot{\vec{x}}(t) = F(t)\vec{x}(t) + G(t)\vec{w}(t) \quad (1)$$

and the measurement equation

$$\vec{y}(t) = H(t)\vec{x}(t) + \vec{v}(t) \quad (2)$$

If  $\vec{w}$  and  $\vec{v}$  are Gaussian random processes, then the system above is known as *Gauss-Markov process*. This process is used to describe a variety of error models of interest. Particularly,

- random constant,
- random walk, and
- first order Markov.

Assuming that the error is due to an unknown constant (i.e. random constant), the following model is used

$$\dot{\vec{x}}(t) = 0 \quad (3)$$

This model is useful in describing effects such as bias stability, scale factor errors, and misalignments. The value can vary from turn-on to turn-on.

An INS system requires the integration of accel and gyro measurements. This leads to the Gauss-Markov process

$$x(t) = \int_0^t w(\nu) d\nu \Rightarrow \dot{x} = w(t)$$

with

$$\mathbb{E}\{w(t)\} = 0 \tag{4}$$

$$\mathbb{E}\{w(t)w(t + \tau)\} = Q\delta(\tau) \tag{5}$$

$$\mathbb{E}\{x(t)\} = 0$$

$$\mathbb{E}\{x^2(t)\} = Qt$$

called random walk.

- In the case of the gyroscope,  $w$  would be in the rate domain and thus has units of  $deg/s$  and the units of  $x(t)$  is degrees.
- $\mathbb{E}\{x^2(t)\} = Qt$  has units of  $deg^2$ .
- Therefore,  $Q$  has the units of  $\frac{deg^2}{s} = \frac{(deg/s)^2}{Hz}$  which is the PSD level of the gyro white noise.
- This matches the units in Eq. 5 (Note that  $\delta(\tau)$  has units of 1/time).
- In simulation the power of the white noise  $w$  that is experienced by the system is  $F_s Q$ , where  $F_s$  is the sampling rate.
- Consequently,  $\sigma_w = \sqrt{F_s Q}$  and has the units of  $deg/s$
- Recall that the gyro angle random walk (ARW)

$$ARW(^{\circ}/\sqrt{h}) = \frac{1}{60} \sqrt{PSD((^{\circ}/h)^2/Hz)} \quad (6)$$

Frequently used to model bias instability (BI)

## State Equation

$$\dot{b}(t) = -\frac{1}{T_c}b(t) + w(t) \quad (7)$$

## Autocorrelation

$$\mathbb{E}\{b(t)b(t + \tau)\} = \sigma_{BI}^2 e^{-|\tau|/T_c} \quad (8)$$

where

$$\mathbb{E}\{w(t)w(t + \tau)\} = Q\delta(t - \tau) \quad (9)$$

$$Q = \frac{2\sigma_{BI}^2}{T_c} \quad (10)$$

and  $T_c$  is the correlation time.

## State Equation

$$b_k = e^{-\frac{1}{T_c} \tau_s} b_{k-1} + w_{k-1} \quad (11)$$

## Covariance Matrix of the discrete Markov Noise

$$Q_d = \sigma_{B_I}^2 [1 - e^{-\frac{2}{T_c} \tau_s}] \quad (12)$$











