EE 565: Position, Navigation, and Timing

Global Navigation Satellite Systems (GNSS)

Part I

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Dead Reckoning vs Position Fixing

- Navigation can be accomplished via “position fixing” or “dead reckoning”
  1. Dead Reckoning — Measures changes in position and/or attitude
     - Inertial sensors provide relative position (and attitude)
  2. Position Fixing — Directly measuring location
     - GPS provides absolute positino (and velocity)
- How does GPS work?
  - Effectively via Multilateration
    - If I can measure my distance to three (or more) satellites at known locations, then, own location can be resolved. *Measure distance via “time-of-flight”*

GNSS

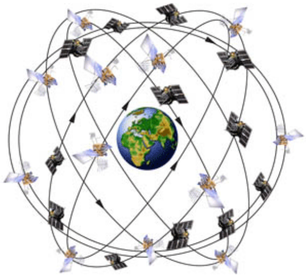
- GNSS — A generic term used to describe these navigation systems that provide a user with 3-D positioning solution using RF ranging of signals transmitted by orbiting satellite
- GNSS examples include
  - NAVSTAR — Navigation by Satellite Ranging and Timing operated by the United States commonly referred to as Global Positing System (GPS)
  - GLONASS — Russian
  - Galileo — European
  - Beidou — China

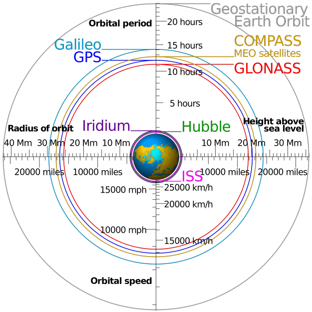
GNSS Architecture

- Space segment (satellites)
- Control segment
- User segment

Space Segment

- Collection of satellites known as *constellation*
- Broadcasts signals to control segement and the users
- Distributed among different medium Earth orbits (MEOs)
- GPS satellites
    - orbit at a radius of 26,580km
    - two orbits per sidereal day





Control Segment

- Consists of
  - monitoring stations — at surveyed locations with synchronized clocks and collects ranging measurements
  - control stations — received data from monitoring stations and calculates corrections
  - uplink stations — sends commands to the satellites.

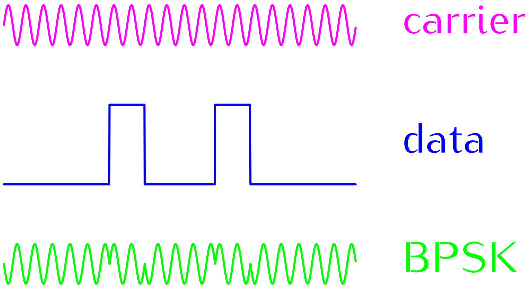
GNSS Signals

In general, a GNSS signal is a carrier with a spreading code modulated using binary phase shift keying (BPSK) given by

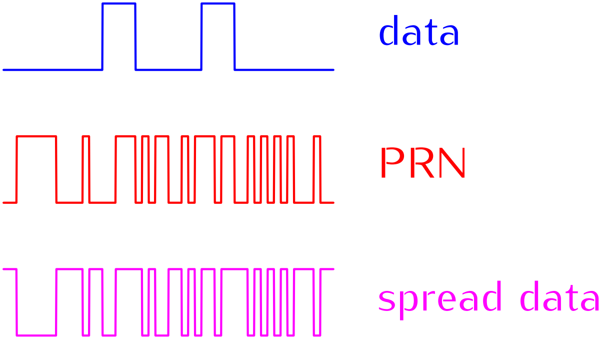
$$s(t) = \sqrt{2P}C(t)D(t)\cos(2\pi f_{ca}t + \phi_0) \tag{1}$$

where P is the signal power, $C(t)$ is the spreading code, $D(t)$ is the data, f_{ca} is the carrier frequency, and ϕ_0 is the phase offset. $C(t)$ and $D(t)$ have ± 1 values.

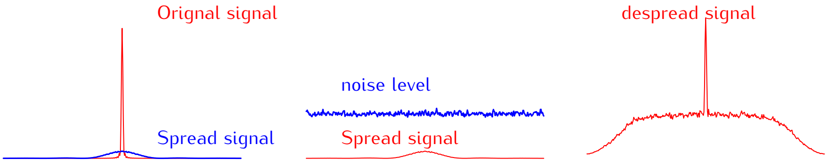
BPSK



Spreading Code



Signal Power



GPS Modulation Schemes

- The GPS employs BPSK modulation at two frequencies
  - L1=1,575.42 MHz
  - L2=1,227.60 MHz
- Two main PRN code
  - C/A: Course acquistion (10-bit 1MHz)
  - P: Precise
    - 40-bit 10MHz
    - Encrypted P(Y) code

Ranging Basics

By determining the phase of the received PRN code the raw pseudo-range to a given satellite is given by

$$\tilde{\rho}^{s}_{a,R} = (\tilde{t}^{s}_{sa} - \tilde{t}^{s}_{st,a})c \tag{2}$$

where $\tilde{t}^{s}_{sa}$ is the transmission time of the signal from the satellite, s, $\tilde{t}^{s}_{st,a}$ is the arrival time at antenna, a, and c is the speed of light.

True Range, LOS and Range Rates

The true range from an antenna a to a satellite s in the ECEF frame is given by

$$\vec{r}_{as} = |\vec{r}^{\,e}_{es}(t^{s}_{st,a}) - \vec{r}^{\,e}_{ea}(t^{s}_{sa,a})| + \delta\rho^{s}_{ie,a} \tag{3}$$

where $\delta\rho^{s}_{ie,a}$ is a correction factor due to rotation of the earth causing Sagnac effect. The line-of-sight unit vector (direction from which a signal arrives at the user antenna) in the ECEF frame is given by

$$\vec{u}^{\,e}_{as} \approx \frac{\vec{r}^{\,e}_{es}(t^{s}_{st,a}) - \vec{r}^{\,e}_{ea}(t^{s}_{sa,a})}{|\vec{r}^{\,e}_{es}(t^{s}_{st,a}) - \vec{r}^{\,e}_{ea}(t^{s}_{sa,a})|} \tag{4}$$

The range rate using ECEF velocities is

$$\dot{\vec{r}}_{as} = (\vec{u}^{\,e}_{as})^{T}(\vec{v}^{\,e}_{es}(t^{s}_{st,a}) - \vec{v}^{\,e}_{es}(t^{s}_{sa,a})) + \delta\dot{\rho}^{s}_{ie,a} \tag{5}$$

Sagnac Correction

The Sagnac correction is approximated as

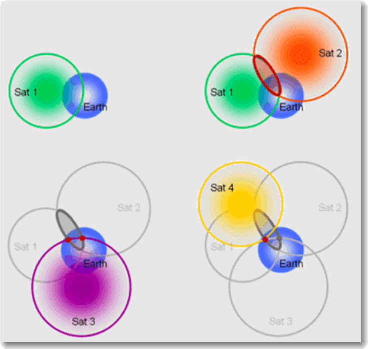
$$\delta\rho^s_{ie,a} \approx \frac{\omega_{ie}}{c}\left[y^e_{es}(t^s_{st,a})x^e_{ea}(t^s_{sa,a}) - x^e_{es}(t^s_{st,a})y^e_{ea}(t^s_{sa,a})\right] \tag{6}$$

and the range-rate Sagnac correction is

$$\delta\dot{\rho}^s_{ie,a} \approx \frac{\omega_{ie}}{c}\begin{pmatrix} v^e_{es,y}(t^s_{st,a})x^e_{ea}(t^s_{sa,a}) + y^e_{es}(t^s_{st,a})v^e_{ea,x}(t^s_{sa,a}) \\ -v^e_{es,x}(t^s_{st,a})y^e_{ea}(t^s_{sa,a}) - x^e_{es}(t^s_{st,a})v^e_{ea,y}(t^s_{sa,a}) \\ 0 \end{pmatrix} \tag{7}$$

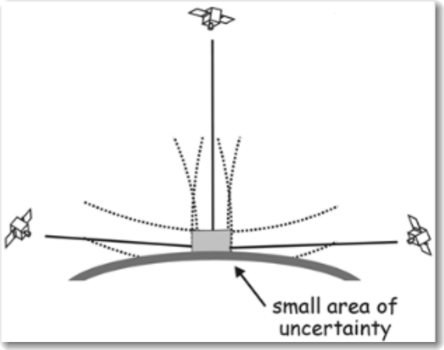
Multilateration

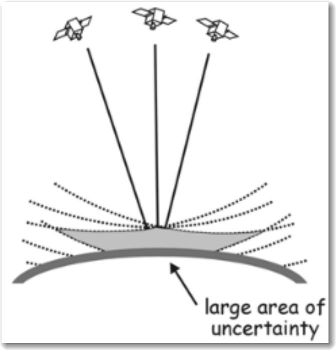
Use the range to multiple satellites to determine the position of the user equipment.



Geometric Dillution of Precision

GNSS solution is affected by the geometry of the satellite constellation observed by the receiver antenna.





Positioning

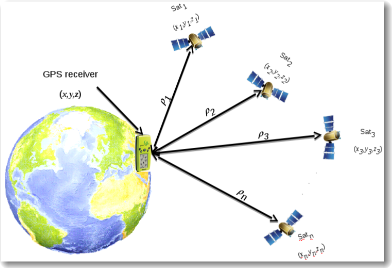
All measurements are in ECEF

$$\rho_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}$$

$$\rho_i^2 = x_i^2 + x^2 - 2x_i x + y_i^2 + y^2 - 2y_i y + z_i^2 + z^2 - 2z_i z$$

$$\rho_i^2 - (x_i^2 + y_i^2 + z_i^2) - (x^2 + y^2 + z^2) = -2x_i x - 2y_i y - 2z_i z$$

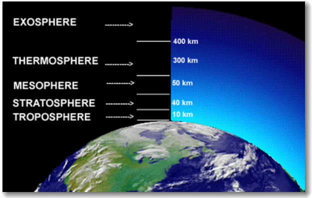
$$\begin{pmatrix} \rho_1^2 - (x_1^2 + y_1^2 + z_1^2) - r_e^2 \\ \rho_2^2 - (x_2^2 + y_2^2 + z_2^2) - r_e^2 \\ \vdots \\ \rho_n^2 - (x_n^2 + y_n^2 + z_n^2) - r_e \end{pmatrix} = \begin{pmatrix} -2x_1 - 2y_1 - 2z_1 \\ -2x_2 - 2y_2 - 2z_2 \\ \vdots \\ -2x_n - 2y_n - 2z_n \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



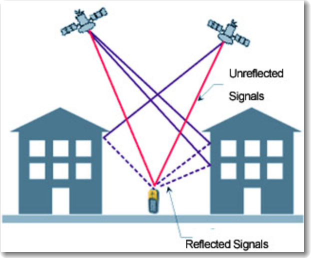
Measurements of pseudorange

In reality there are errors in the propagation model used for the signal due to ionosphere and the troposphere. In addition there are clock errors both at the satellite and the receiver. Consquently, the pseudorange measurement is given by

$$\begin{aligned}\rho_{measured} =& \rho_{true} + \epsilon_{ionospheric} + \epsilon_{tropospheric} + \epsilon_{ephemeris} + \\ & \epsilon_{satellite\ clock} + \epsilon_{receiver\ clock} + \epsilon_{multipath}\end{aligned} \tag{8}$$



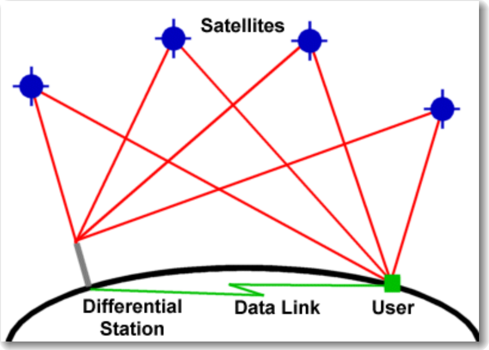
www.intellego.fr (blog by manumanu)



http://www.engineeringsall.com/sources-of-errors-in-gps/

Error Mitigation Techniques — Differential GPS

- Measure pseudorange error at surveyed locations
- Subtract error at the user equipment before calculating position



Error Mitigation Techniques — WAAS GPS

Wide Area Augmentation System

- Provide corrections based on user position
- Assumes atomospheric errors are locally correlated.