

Lecture

Navigation Mathematics: Coordinate Frames

EE 565: Position, Navigation and Timing

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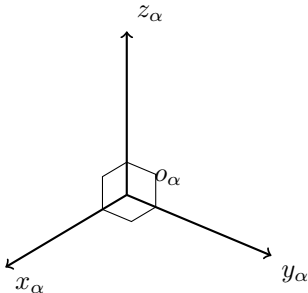
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1 Coordinate Frames

Coordinate Frames

Right-hand coordinate frame α has

1. origin o_α at which frame is located, and
2. orthonormal vectors $x_\alpha, y_\alpha, z_\alpha$ that serve as axes and indicate positive directions.



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Coordinate Frames

This definition implies

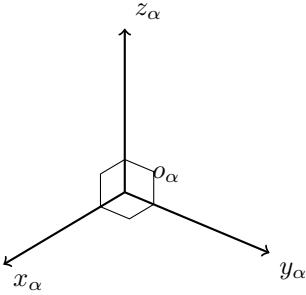
$$x_\alpha \cdot x_\alpha = y_\alpha \cdot y_\alpha = z_\alpha \cdot z_\alpha = 1$$

$$x_\alpha \cdot y_\alpha = y_\alpha \cdot z_\alpha = z_\alpha \cdot x_\alpha = 0$$

$$x_\alpha \times y_\alpha = z_\alpha$$

$$y_\alpha \times z_\alpha = x_\alpha$$

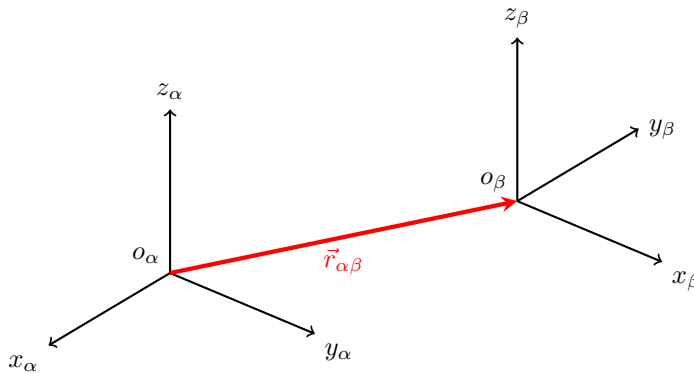
$$z_\alpha \times x_\alpha = y_\alpha$$



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Coordinate Frames

Coordinate frames used as means to describe position and orientation/attitude of one frame with respect to another.



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2 Earth-Centered Inertial (ECI) Frame

Earth-Centered Inertial (ECI) Frame

ECI Frame

- defined as an inertial frame, i.e., it is assumed not to accelerate or rotate with respect to the universe
 - effects of earth’s orbit around sun and motion of the galaxy are very small (smaller than can be measured with inertial sensors) and neglected
 - ECI will be attached to earth, but won’t spin with earth
- inertial sensors measure “inertial” motion relative to ECI frame
 - Gyroscopes measure rate of change of orientation
 - Accelerometers measure linear acceleration
- referred to as *i*-frame

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ECI Frame

- origin o_i of ECI is located near the center of mass (center of ellipsoidal representation) of the earth
- z_i -axis points along the nominal axis of rotation of the earth
 - true north **not** magnetic north!
 - spin axis moves in circular path with radius of 15 meters, which we’ll neglect and use average value

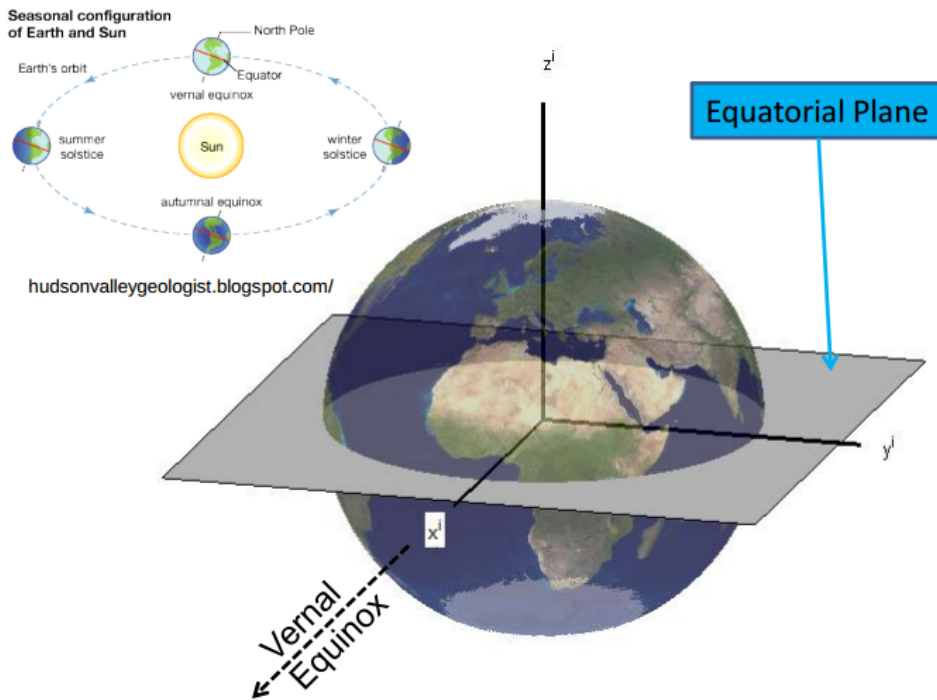
- x_i -axis lies in the equatorial plane and points from the earth to the sun at the vernal (spring) equinox
 - defined by the intersection (a line) of the equatorial plane and the earth-sun orbital plane
- y_i -axis chosen to complete right hand coordinate system (90° ahead of x_i in direction of earth's rotation)

*The ECI coordinate frame does **not** rotate with the earth*

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ECI Frame

- o_i at earth's center
- z_i -axis points along the earth's axis of rotation
- x_i -axis points towards sun at vernal (spring) equinox
- y_i -axis completes a right hand coordinate system



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3 Earth-Centered Earth-Fixed (ECEF) Frame

Earth-Centered Earth-Fixed (ECEF) Frame

ECEF Frame

- **not** an inertial frame
- fixed with respect to the earth, i.e., attached to the earth and spins with earth
- referred to as e -frame

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ECEF Frame

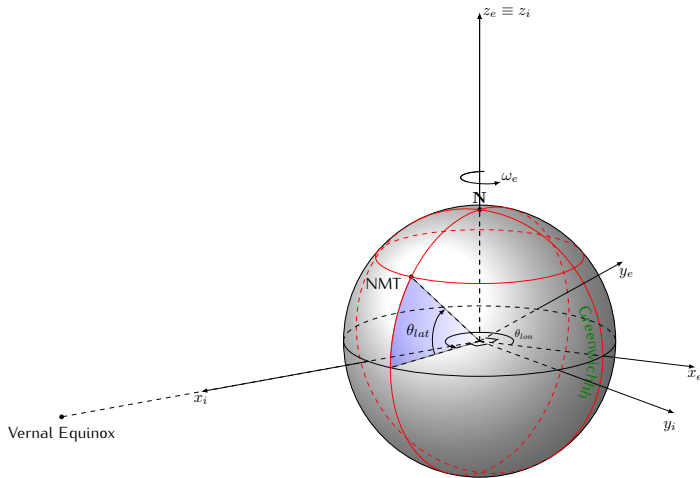
- origin o_e is located (nearly) at the center of the mass of the earth (co-located with ECI's o_i)
- z_e -axis points along the nominal axis of earth's rotation (same as ECI's z_i)
- x_e -axis lies at the intersection of the equatorial plane and the reference meridian plane (i.e., Greenwich/Prime Meridian)

- tied to concept of latitude and longitude
- x_e points from o_e towards 0° longitude and 0° latitude (a little west of central Africa)
- y_e -axis is chosen to complete right hand coordinate system

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ECEF Frame

- z_e -axis points along axis of earth's rotation
- x_e -axis points towards zero latitude and zero longitude
- y_e -axis completes right hand coordinate system
- NMT's (lat, long) $\approx (34.07^\circ, -106.9^\circ) = (34.07^\circ, 253.1^\circ)$



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4 Local Navigation (Nav) Frame

Local Navigation (Nav) Frame

Nav Frame

- typically **not** fixed with respect to the earth, i.e., free to move, but has specified orientation
- also called geodetic, geographic, locally level, or tangential frame
- referred to as n -frame

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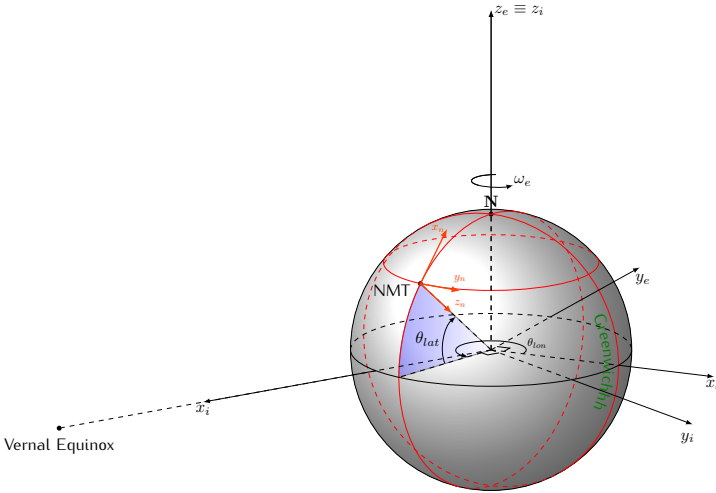
Nav Frame

- origin o_n is located at the center of mass of the body (e.g., air, land or sea vehicle) of interest
- z_n -axis points "down" normal to the earth's surface (approximately towards the center of the earth)
- $x_n - y_n$ axes then constrained to lie in plane locally-level (tangential) to the earth's surface
 - x_n -axis points to the north pole
 - y_n -axis is chosen to complete right hand coordinate system
- frame's configuration is often referred to as the NED frame
 - $x_n \rightarrow$ North, $y_n \rightarrow$ East, and $z_n \rightarrow$ Down

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Nav Frame

- o_n on (potentially moving) body
- x_n -axis points north
- y_n -axis points east
- z_n -axis points "down"



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5 Body Frame

Body Frame

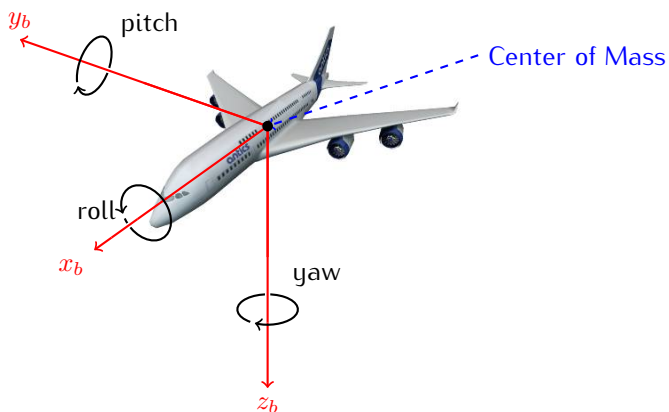
Body Frame

- attached to moving body (e.g., land, air or sea vehicle) and moves (position and orientation/attitude) with body
- origin o_b located at the center of mass of the body (co-located with Nav frame's o_n)
- x_b -axis points "forward" wrt moving body
- z_b -axis points loosely "down"
 - varies with the roll/pitch of the vehicle
- y_b -axis chosen to complete right hand coordinate system
- referred to as b -frame

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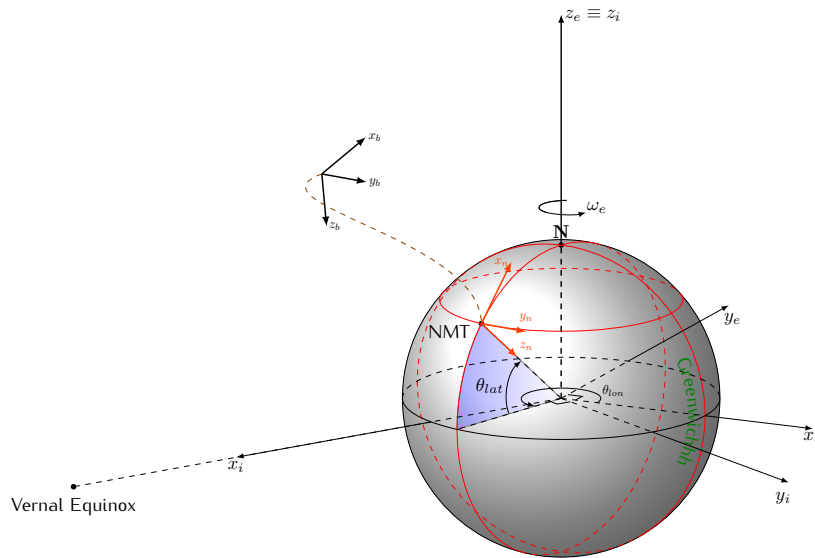
Body Frame

- body frame is fixed with respect to the vehicle
- x_b "forward"
- z_b "down"
- y_b completes right hand coordinate system ("right")



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Body Frame



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6 Other Frames

Other Frames

- Wander Azimuth Frame (alternative to the Nav frame)
 - does not always point north (x - and y - axes displaced from north and east by an angle) to avoid numerical stability problems near the poles
- Other locally level frames
 - Tangential Frame
 - * typically, refers to another type of the ECEF frame fixed to the Earth's surface (not moving like the n -frame)
 - Computer Frame
 - * virtual coordinate frame that represents where we think we are

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The End

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