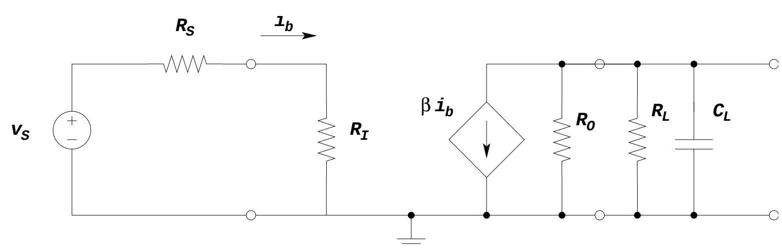
# EE 321 - Exam 1

## September 30, 2002

Name: ______________________________________________

Closed book. One page of notes and a calculator are allowed. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work.

1. Consider the following amplifier:




where $R_S = 1$ kΩ, $R_I = 2$ kΩ, $R_O = 90$ kΩ, $R_L = 10$ kΩ, $C_L = 0.01$ $\mu$F and $\beta = 90$.

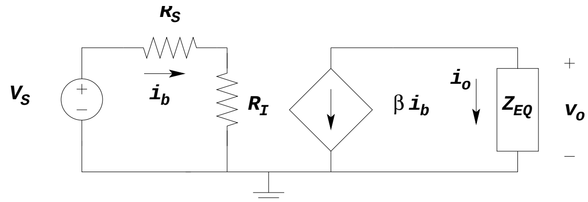
(a) Find the transfer function $T(s) = v_O(s)/v_S(s)$.

(b) Is the amplifier a low-pass or high-pass amplifier.

(c) Find the 3 dB frequency of the amplifier.

(d) Sketch a Bode plot for $|T(\omega)|$ (in dB) vs. $\omega$ (on a log scale).

(a) $R_o$, $R_L$ and $C_L$ are all in parallel so the voltage across all of them are the same. Combine $R_o$, $R_L$ and $C_L$ into a single equivalent impedance $Z_{EQ}$:




$$Z_{EQ} = \frac{1}{\frac{1}{R_o} + \frac{1}{R_L} + C_L} = \frac{1}{\frac{1}{R_o||R_L} + C_L s} = \frac{R_o||R_L}{1 + (R_o||R_L)C_L s}$$

Now $v_o = i_o Z_{EQ}$, and $i_o = -\beta i_b$, and $i_b = \frac{v_S}{R_S + R_I}$, so

$$v_o = -\beta i_b Z_{EQ} = -\beta \frac{v_S}{R_S + R_I} \frac{R_o||R_L}{1 + (R_o||R_L)C_L s} = \frac{-\beta v_S (R_o||R_L)/(R_S + R_I)}{1 + (R_o||R_L)C_L s}$$

and

$$T(s) = \frac{V_o(s)}{V_S(s)} = \frac{-\beta(R_o||R_L)/(R_S + R_I)}{1 + (R_o||R_L)C_L s}$$

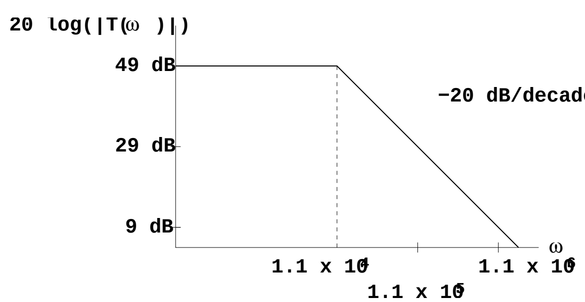
$R_o||R_L = \frac{R_o R_L}{R_o + R_L} = 9\text{k}\Omega$, so

$$T(s) = \frac{-90(9\text{k}\Omega)/(1\text{k}\Omega + 2\text{k}\Omega)}{1 + (9\text{k}\Omega)(10\mu\text{F})s} = \frac{-270}{1 + 9 \times 10^{-5} s} = \frac{K}{1 + s/\omega_o}$$

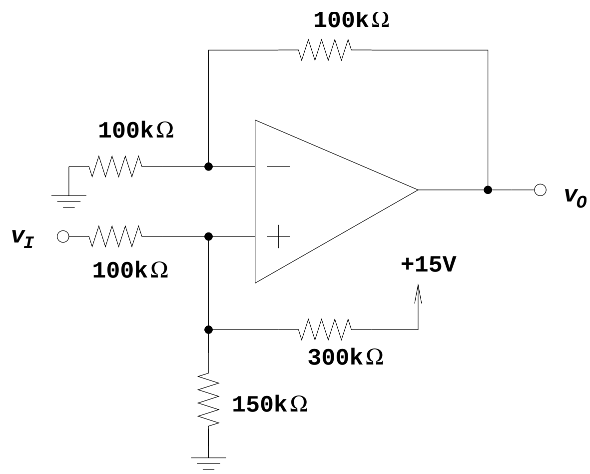
(b) This is the form of a low-pass filter. To verify, note that at high frequencies, the capacitor acts as a short circuit, so $v_o = 0$, and at low frequencies, the the capacitor acts as an open circuit, so $v_o = -\beta i_b (R_o||R_L)$ = -270.

(c) The break frequency is $\omega_o$ in the above equation. Also, $C_L$ is in parallel with $R_o$ and $R_L$, so $\omega_o = 1/((R_o||R_L)C_L)$. $\omega_o = 1/9 \times 10^{-5} = 1.11 \times 10^4$ rad/sec, or $f_o = \omega_o/(2\pi) =$ 1.77 kHz.

(d) The gain at DC is $20\log(|K|) = 20\log(270) = 49$ dB. The break frequency is $1.11 \times 10^4$ rad/sec. The single time constant low pass filter falls off at 20 dB/decade for frequencies higher than $\omega_o$.



2. The figure below shows an op-amp circuit. The op-amp uses ±15 V power supply voltages, and the output saturation levels are ±14 V.




(a) Find the transfer function $v_O/v_I$.

(b) Find the range of input voltages for which the output is not saturated.

(a) Note that this is a a non-inverting amplifier, which amplifies the voltage at the non-inverting input:

$$v_o = (1 + R_2/R_1)v_+ = 2v_+$$

Now need to find $v_+$. Do this two different ways:

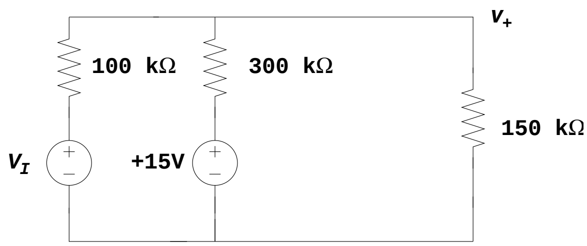
- Do nodal analysis at $v_+$:

$$\frac{v_+ - v_I}{100\text{k}\Omega} + \frac{v_+ - 15\text{V}}{300\text{k}\Omega} + \frac{v_+ - 0}{150\text{k}\Omega} = 0$$

$$3(v_+ - v_I) + (v_+ - 15\text{V}) + 2(v_+ - 0) = 0$$

$$v_+ = \frac{v_I}{2} + 2.5\text{V}$$

- Find $v_+$ by superposition. We can redraw the input to look like this:




  - First turn off the +15 V supply. $v_+$ can be found by a voltage divider of $v_I$ between 150 kΩ|| 300 kΩ and 100 kΩ:

$$v_+ = \frac{300\text{k}||150\text{k}}{300\text{k}||150\text{k} + 100\text{k}} v_I = \frac{100\text{k}}{100\text{k} + 100\text{k}} v_I = \frac{v_I}{2}$$

  - Next turn off $v_I$. $v_+$ can be found by a voltage divider of +15 V between 150 kΩ!! 100 kΩ and 300 kΩ:

$$v_+ = \frac{100\text{k}||150\text{k}}{100\text{k}||150\text{k} + 300\text{k}} 15\text{V} = \frac{60\text{k}}{60\text{k} + 300\text{k}} 15\text{V} = 2.5\text{V}$$

  - The total voltage at $v_+$ is the sum of the above:

$$v_+ = \frac{v_I}{2} + 2.5\text{V}$$

Therefore, $v_o = 2v_+ = v_I + 5\text{V}$

(b) The amplifier saturates at ±14 V. Note that $v_o = v_I + 5$ V.

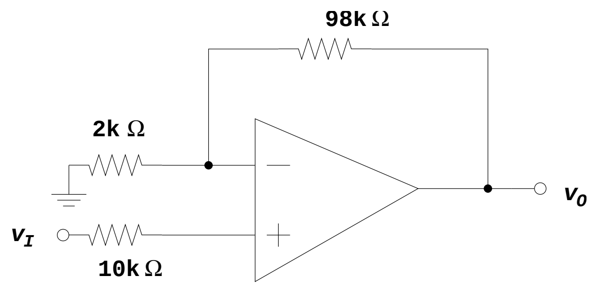
$$-14\text{V} \le v_o \le 14\text{V}$$

$$-14\text{V} \le v_I + 5\text{V} \le 14\text{V}$$

$$-19\text{V} \le v_I \le 9\text{V}$$

Thus, the range for which the output is not saturated is -19 V to +9 V.

3. Consider the op amp circuit below. For parts a) through c), consider the op amp to be ideal.




   (a) Find the gain $v_O/v_I$.

   (b) What is the input resistance $R_{in}$?

   (c) What is the output resistance $R_{out}$?

   For the rest of this problem, assume the op-amp to be non-ideal. It is internally compensated (i.e., gain vs. frequency plot is that of a single time constant low pass circuit). In the lab, the op-amp was measured to have a DC open-loop gain of 120 dB. The open-loop gain at 10 kHz was measured to be 60 dB. The slew rate is 2 V/$\mu$s, and the saturation limits are $\pm 14$ V. The input bias current is 10 nA, and the input offset voltage is 1 mV.

   (d) Find the unity gain bandwidth of the op amp.

   (e) Find the (open loop) 3 dB frequency of the op amp.

   (f) Find the full power bandwidth of the circuit.

   (g) Find the 3 dB frequency of the (closed-loop) amplifier circuit.

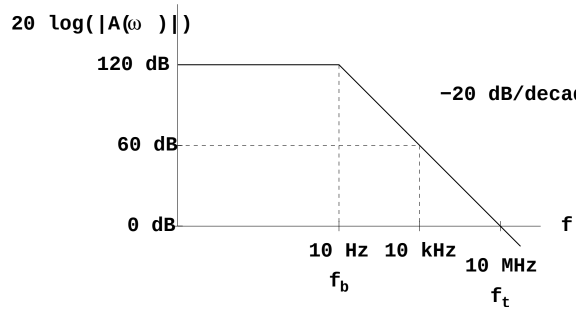
   (h) Find the output voltage when the input voltage is 0.

(a) This is a non-inverting amplifier. $v_O/v_I = 1 + R_2/R_1 = 1 + 98k/2k = 50$

(b) For an ideal op-amp, no current flows into the + input. Thus $i_{in} = 0$, so $R_{in} = v_I/i_{in} = \infty$

(c) The output resistance of an ideal op amp is 0, so $R_{out} = 0$.

(d) The open-loop gain at 10 kHz is 60 dB. The gain is falling off at a rate of 20 dB/decade, so an increase of 3 decades will decrease the gain to 0 dB (0 dB is a gain of 1, i.e., unity gain). Thus the unity gain bandwidth is $f_t = 10$ kHz $\times 10^3 = 10$ MHz.




(e) The open-loop gain at 10 kHz is 60 dB. The gain is falling off at a rate of 20 dB/decade, so a decrease of 3 decades will increase the gain to its value of 120 dB at DC. This will be the break frequency, and the gain at this frequency will be 3 dB below the DC gain. Thus the open-loop 3 dB frequency is $f_b = 10$ kHz $\times 10^{-3} = 10$ Hz.

Or, use the equation $f_t = A_o f_b$. Note that 20 $\log(A_o) = 120$ dB, so $A_o = 10^{120/20} = 10^6$ V/V.

$$f_b = \frac{f_t}{A_o} = \frac{10^7\text{Hz}}{10^6\text{V/V}} = 10\text{Hz}$$

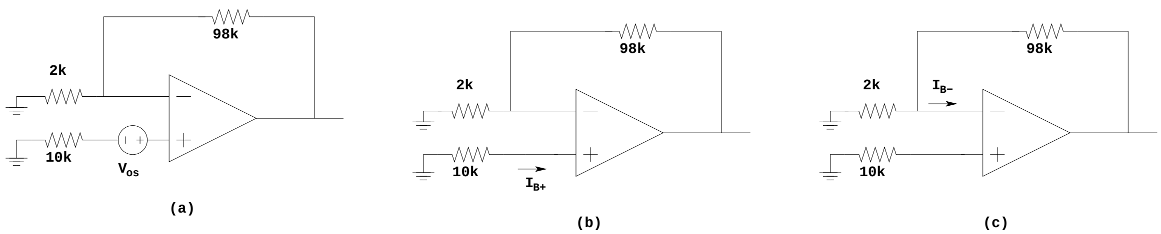
(f) The full-power bandwidth is:

$$f_M = \frac{\text{SR}}{2\pi V o\text{max}} = \frac{2\text{V}/\mu\text{s}}{2\pi 14\text{V}} = \frac{2\times 10^6\text{V/s}}{2\pi 14\text{V}} = 22.7\text{kHz}$$

(g) The 3 dB frequency of a closed-loop amplifier is given by

$$f_{3\text{dB}} = \frac{f_t}{1 + R_2/R_1} = \frac{10\text{MHz}}{1 + 98\text{k}/2\text{k}} = 200\text{kHz}$$

(h) To see effects of offset voltage and bias current, connect $v_I$ to ground. Consider the offset voltage $V_{OS}$ and the bias currents $I_{B+}$ and $I_{B-}$ separately. See figure:




- Figure (a). No current flows into $v_+$, so there is no voltage drop across the 10 k$\Omega$ resistor, and $v_+ = V_{OS}$. The offset voltage $V_{OS}$ is amplified by the gain of the non-inverting amplifier, so $v_o = 50V_{OS} = 50$ mV.
- Figure (b). $I_{B+}$ will flow through the 10 k$\Omega$ resistor, so $v_+$ = -10 nA $\times$ 10 k$\Omega$ = -0.1 mV. This will be amplified by the non-inverting amplifier, so $v_o = 50\times$ (-0.1 mV) = -5 mV.
- Figure (c). $v_- = v_+ = 0$. $I_{B-}$ will flow into (-) terminal. Because $v_- = 0$ V, no current will flow through the 2 k$\Omega$ resistor. All of $I_{B-}$ flows through the 98 k$\Omega$ resistor, so $v_o$ = 98k$\Omega\times$ 10 nA = 0.98 mV $\approx$ 1 mV.

Note that $I_{B+}$ and $I_{B-}$ will always flow in the same direction, so the total output voltage from the input bias currents will be $v_o$ = -5 mV + 1 mV = -4 mV.

The total output voltage will be the sum of the output voltages from the offset voltage and the bias currents. We don't know if the offset voltage is positive or negative, or if the bias current is positive or negative. The maximum output voltage will the be sum of the absolute values of the two, or $|v_o|_{\max} = |\pm 50 \text{ mV}| + |\pm 4 \text{ mV}| = 54$ mV.