Solutions for Homework #23

Chapter 3:

11. Consider an analog repeater system in which the signal has power σ_x^2 and each stage adds noise with power σ_n^2. For simplicity assume that each repeater recovers the original signal without distortion but that the noise accumulates. Find the SNR after n repeater links. Write the expression in decibels: SNR dB = 10 $\log_{10}$SNR.

Solution:

After n stages, the signal power is $\sigma_x^2$and the noise power is $n\sigma_n^2$, so the SNR is:

SNR dB = $10 \log_{10}\sigma_x^2/n\sigma_n^2 = 10 \log_{10}\sigma_x^2/\sigma_n^2 + 10 \log_{10}1/n = 10 \log_{10}\sigma_x^2/\sigma_n^2 - 10 \log_{10}n$

12. Suppose that a link between two telephone offices has 50 repeaters. Suppose that the probability that a repeater fails during a year is 0.01, and that repeaters fail independently of each other.

a. What is the probability that the link does not fail at all during one year?

Let p be the probability that a repeater fails during a year, then $1 - p$ is the probability that it does not fail, and the probability that all 50 repeaters do not fail is $(1 - .01)^{50} \approx e^{-50(.01)}$ = 0.605 where we have used the approximation $(1 - p)^n \approx e^{-np}$ which is valid for large n and small p.

b. Repeat (a) with 10 repeaters; with 1 repeater.

The probability that all 10 repeaters do not fail is $(1 - .01)^{10} \approx e^{-10(.01)}$ = 0.905, and the probability that a single repeater does not fail is 0.99.

The moral of the calculations is that a system that requires the functioning of a large number of relatively reliable components may be fairly unreliable. In terms of repeaters, this implies that minimizing the number of repeaters needed in a link is important from the point of view of reliability. Of course this also reduces the cost expended to install and maintain the repeaters.

13. Suppose that a signal has twice the power as a noise signal that is added to it. Find the SNR in decibels. Repeat if the signal has 10 times the noise power? 2^n times the noise power? 10^k times the noise power?

Solution:

SNR dB = $10 \log_{10}\sigma_x^2/\sigma_n^2 = 10 \log_{10}2 = 3.01$ dB

SNR dB = $10 \log_{10}10 = 10$ dB

SNR dB = $10 \log_{10}2^n = = 10n \log_{10}2 = 3.01n$ dB

SNR dB = $10 \log_{10}10^k = 10k$ dB

25. A square periodic signal is represented as the following sum of sinusoids:

$$g(t) = \frac{2}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}\cos(2k+1)\pi t$$

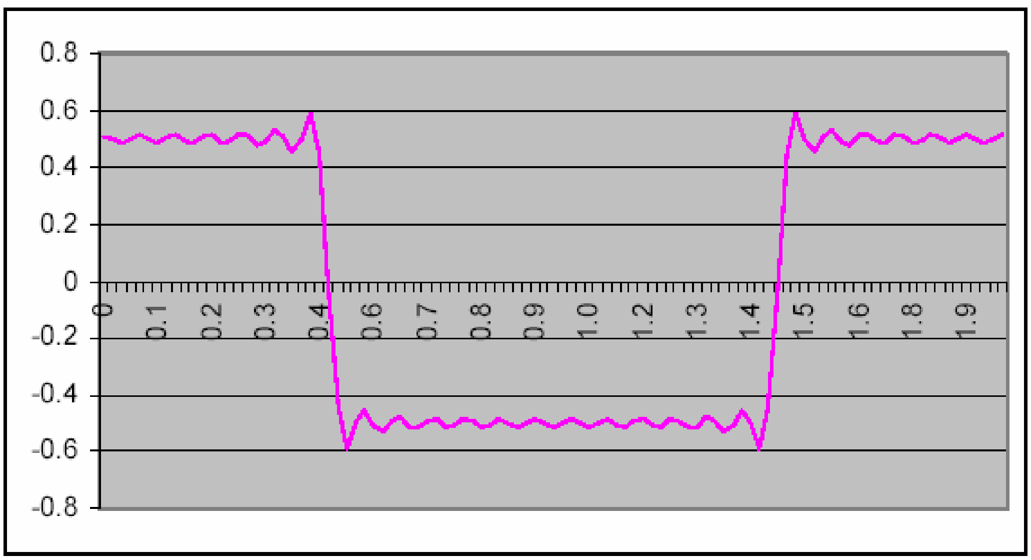
a. Suppose that the signal is applied to an ideal low-pass filter with bandwidth 15 Hz. Plot the output from the low-pass filter and compare to the original signal. Repeat for 5 Hz; for 3 Hz. What happens as W increases?

If we expand the above series to obtain the first few terms of $g(t)$, we get:

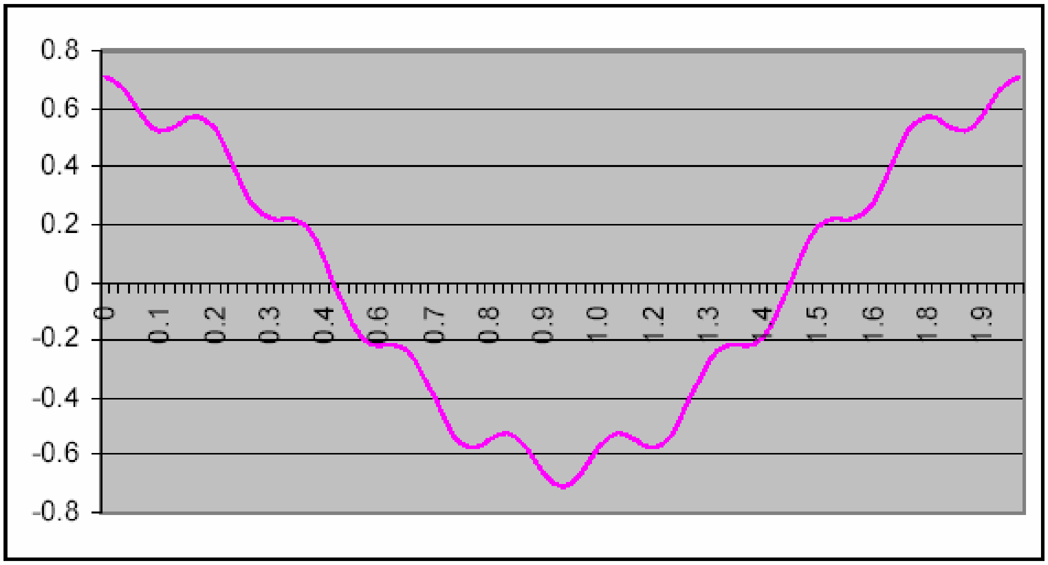
$g(t) = 2/\pi \{ \cos\pi t - (1/3)\cos 3\pi t + (1/5)\cos 5\pi t - (1/7)\cos 7\pi t + (1/9)\cos 9\pi t - \ldots\}$

So frequencies of the components in Hz are 1/2, 3/2, 5/2, 7/2, 9/2, and so on.

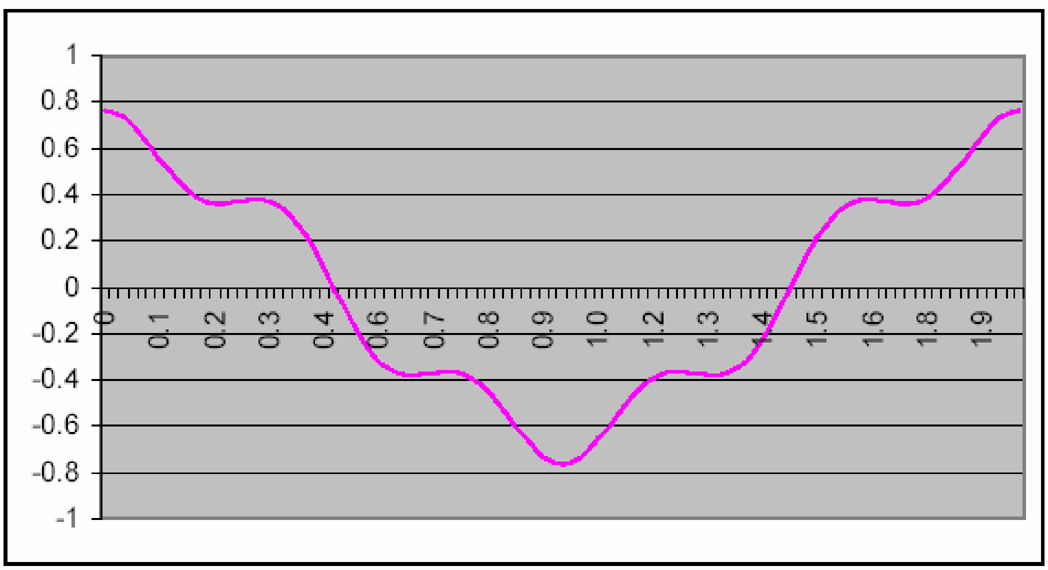
If the signal is applied to a low pass filter with bandwidth 15 Hz, then the output will consist of the sum of all components up to frequency 14.5 Hz, that is, up to component $(2k + 1)/2 = 14.5$, which gives $k = 14$. The resulting signal is shown below.



If the lowpass filter has bandwidth 5 Hz, then the first five components are passed (k=4). The resulting signal is shown below:

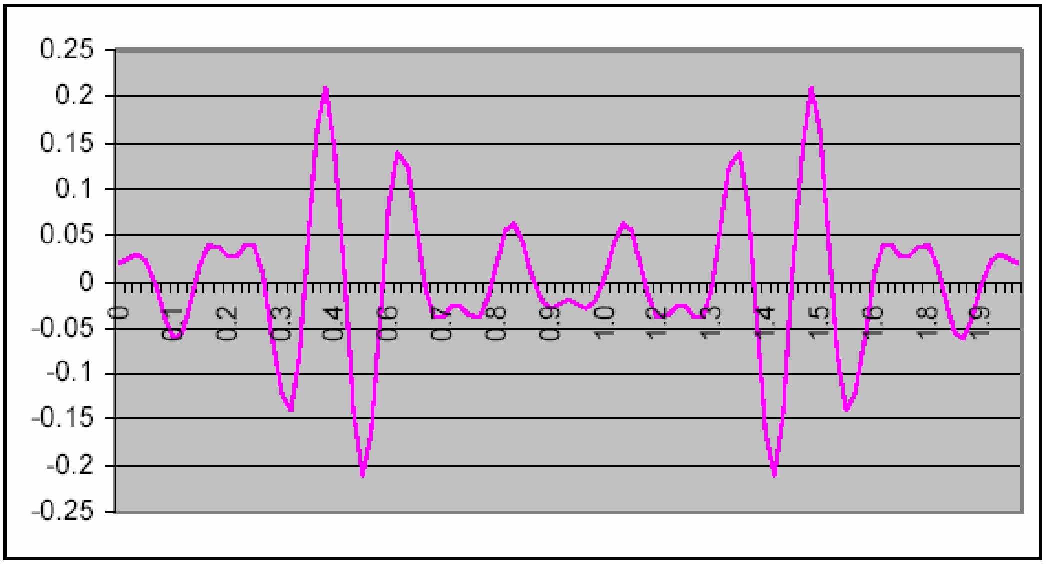


If the lowpass filter has bandwidth 3 Hz, then the first three components are passed (k=2). The resulting signal is shown below:



b. Suppose that the signal is applied to a bandpass filter which passes frequencies from 5 to 9 Hz. Plot the output from the filter and compare to the original signal.

In this case the filter passes the components corresponding to k = 5, 6, 7, and 8. The resulting signal is shown below:



While the three signals in part (a) are seen to be converging to a given signal, the signal obtained in part (b) shows no clear relationship to the signals in part (a)

27. A 10 kHz baseband channel is used by a digital transmission system. Ideal pulses are sent at the Nyquist rate and the pulses can take 16 levels. What is the bit rate of the system?

Solution:

Nyquist pulses can be sent over this channel at a rate of 20000 pulses per second. Each pulse carries $\log_2 16 = 4$ bits of information, so the bit rate is 80000 bits per second.

29. What is the maximum reliable bit rate possible over a telephone channel with the following parameters?

Solutions follow questions:

a. $W = 2.4$ kHz SNR = 20 dB

An SNR of 20 dB corresponds to a value of 100. The channel capacity formula then gives

$C = 2400 \log_2 (1 + 100) = 15979$ bps.

b. $W = 2.4$ kHz SNR = 40 dB

$C = 2400 \log_2 (1 + 10000) = 31890$ bps.

c. $W = 3.0$ kHz SNR = 20 dB

$C = 3000 \log_2 (1 + 100) = 19974$ bps.

d. $W = 3.0$ kHz SNR = 40 dB

$C = 3000 \log_2(1 + 10000) = 39863$ bps.

30. Suppose we wish to transmit at a rate of 64 kbps over a 3 kHz telephone channel. What is the minimum SNR required to accomplish this?

Solution:

We know that R = 64 kbps and W = 3 kHz. What we need to find is SNR_{min}. The channel capacity is:

$C = W \log_2(1 + SNR)$, $C \geq C_{min} = 64$ kbps

$C_{min} = W \log_2(1 + SNR_{min}) \Rightarrow \log_2(1 + SNR_{min}) = 64/3 \Rightarrow 1 + SNR_{min} = 2^{64/3}$

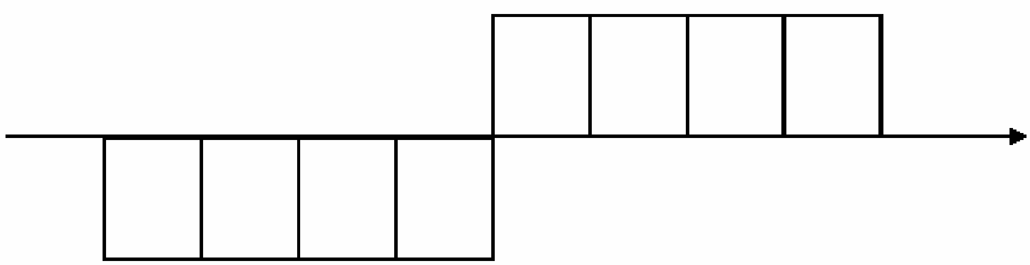
$\Rightarrow SNR_{min} = 2.64 \times 10^6$

in dB: $SNR_{min} = 10 \log_{10}(2.64 \times 10^6) = 64.2$ dB $\therefore$ a very clean channel

32. Most digital transmission systems are "self-clocking" in that they derive the bit synchronization from the signal itself. To do this the systems use the transitions between positive and negative voltage levels. These transitions help define the boundaries of the bit intervals.

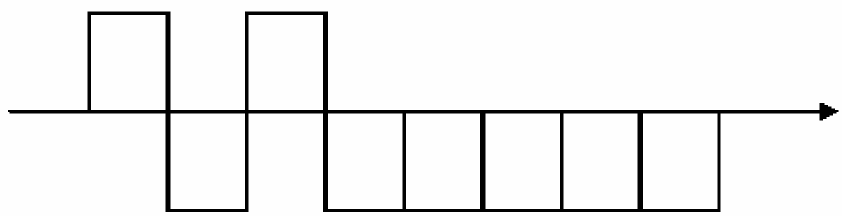
Solutions follow questions:

a. The nonreturn-to-zero (NRZ) signaling method transmits a 0 with a +1 voltage of duration T, and a 1 with a –1 voltage of duration T. Plot the signal for the sequence n consecutive 1s followed by n consecutive 0s. Explain why this code has a synchronization problem.



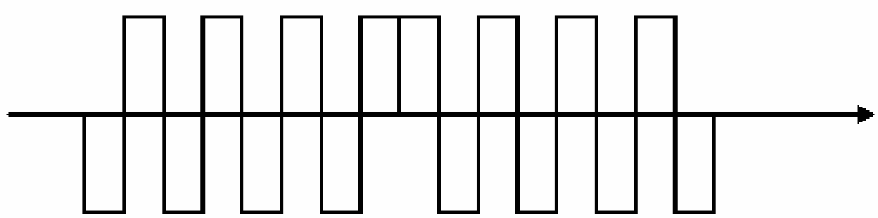
The above figure shows a sequence of 4 1s followed by 4 0s. A long sequence of 1s or a long sequence of 0s produces a long period during which there is no change in the signal level. Consequently, there are no transitions ("zero crossings") that help a synchronization circuit determine where the boundary of each signaling interval is located.

b. In differential coding the sequence of 0s and 1s induces changes in the polarity of the signal; a binary 0 results in no change in polarity, and a binary 1 results in a change in polarity. Repeat part (a). Does this scheme have a synchronization problem?



The occurrence of a "1" induces a transition and helps synchronization. However sequences of "0s" still result in periods with no transitions.

c. The Manchester signaling method transmits a 0 as a +1 voltage for $T/2$ seconds followed by a –1 for $T/2$ seconds; a 1 is transmitted as a –1 voltage for $T/2$ seconds followed by a +1 for $T/2$ seconds. Repeat part (a) and explain how the synchronization problem has been addressed. What is the cost in bandwidth in going from NRZ to Manchester coding?



Every T-second interval now has a transition in the middle, so synchronization is much simpler. However, the bandwidth of the signal is doubled, as pulses now are essentially half as wide, that is, $T/2$ seconds.

33. Consider a baseband transmission channel with a bandwidth of 10 MHz. What bit rates can be supported by the bipolar line code and by the Manchester line code?

Solution:

From Figure 3.26 we see that a bipolar code with pulses T-seconds wide occupies a bandwidth of $W = 1/T$ Hz. Therefore a 10 MHz bandwidth allows a signaling rate of 10 megabits/second.

From the figure it can also be seen that a Manchester code occupies twice the bandwidth. Hence a 10 MHz bandwidth allows a signaling rate of 5 megabits/second.

36. Suppose a CATV system uses coaxial cable to carry 100 channels, each of 6 MHz bandwidth. Suppose that QAM modulation is used.

QAM modulation with 2^m point $\Rightarrow$ m bits/pulse

T = pulse duration = $1/W \Rightarrow$ pulse rate = baud rate = $W = 6 \times 10^6$

a. What is the bit rate/channel if a four-point constellation is used? eight-point?

~~$R = mW$, if $m = 4 \Rightarrow R = 24$ Mbps. If $m = 8$, then $R = 48$ Mbps~~

b. Suppose a digital TV signal requires 4 Mbps. How many digital TV signals can each channel handle for the two cases in part (a)?

R_{DTV} = 4 Mbps $\Rightarrow N_{DTV} = R/R_{DTV}$, where N is the number of digital TV signals per channel

~~If $m = 4 \Rightarrow N_{DTV} = 6$; if $m = 8 \Rightarrow N_{DTV} = 12$.~~

(a) R=mW; if m = 2 (4-point), then R = 12 Mbps. If m =3 (8-point), then R= 18 Mbps

(b) If m= 2 (4-point), then N_{DTV} = 3; If m=3 (4-point), then N_{DTV}=4.5 (4 signals in practice)