

EE212L: Butterworth Filters

The Butterworth filter is a commonly used low-pass filter as it has a maximally flat magnitude in the passband and a generally linear phase response. The filter is designed such that the gain of the passband frequencies is flat, resulting in no gain ripples, and the poles are inherently located solely in the left hand quadrants of the s -plane. By increasing the order of the filter one can increase the rate at which the magnitude transitions from the passband to the stopband, acting more like an ideal low pass filter. The Butterworth filter, and many other filter types, can be achieved by utilizing the Sallen-Key circuit (see Figure 1), as it requires a simple op-amp configuration with calculable component values.

Objective: For this lab you will be designing, implementing, and testing an analog Butterworth filter. In order to do so, you must understand the generic Sallen-Key filter topology and many common filter terms such as passband gain and stopband attenuation. Make sure the entire process is well documented, as you will be required to write up a formal report detailing your design procedure and lab testing.

Prelab: Design a low-pass Butterworth filter that will pass frequencies of up to 1000 rad/s with a gain of 6dB and a maximum passband deviation of 3dB while attenuating frequencies beyond 4000 rad/s with less than -20dB gain. The general form of a Butterworth transfer function can be achieved by following the steps outlined below.

The maximum allowable passband deviation, A_{max} in dB, can be related to the associated parameter ϵ as seen below in equation 1.

$$A_{max} = 20 \log \sqrt{1 + \epsilon^2} \quad \text{Eq. (1)}$$

Similarly, at the edge of the stopband the signal attenuation (in dB) is given in equation 2 where ω_s and ω_p are the stop and passband frequencies respectively. This can be used to determine the minimum filter order, N , where N is rounded to the nearest integer value.

$$A(\omega_s) = -10 \log [1 + \epsilon^2 (\omega_s / \omega_p)^{2N}] \text{Eq. (2)}$$

The natural modes, or poles, of the resulting N th-order Butterworth filter can then easily be determined graphically. The first pole, p_1 , will be located at an angle of $\pi/2N$ to the left of the Imaginary axis in the s -plane. Each pole afterwards (p_2, p_3, \dots, p_N) is spaced by an equal angle of π/N from the first. The radial distance from the origin of the pole can then be represented as

$$\omega_0 = \omega_p \left(\frac{1}{\epsilon} \right)^{\frac{1}{N}}.$$

The transfer function of the Butterworth filter is then obtained as seen in equation 3, where K is a constant equivalent to the DC gain of the filter. The s term can be substituted with the

common expression $j\omega$. This can then be compared to the transfer function of the desired topology to determine component values.

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)} \quad \text{Eq. (3)}$$

1. Sketch the magnitude response that meets the given requirements.
2. Determine N, the minimum order of the filter required to achieve the specified filter.
3. Sketch the pole placement on the s plane. Make sure to include the radial distance from the origin.
4. Determine the transfer function of the Butterworth filter that meets the transmission specifications.
5. Find the transfer function of the generic Sallen-Key filter (as seen in Figure 1). **Hint:** Google is your friend. Often times the components R_A & R_B are left out of the generic form, but can simply be used to scale the DC gain of the filter.
6. Find component values for the Sallen-Key circuit that meet the given filter specifications.

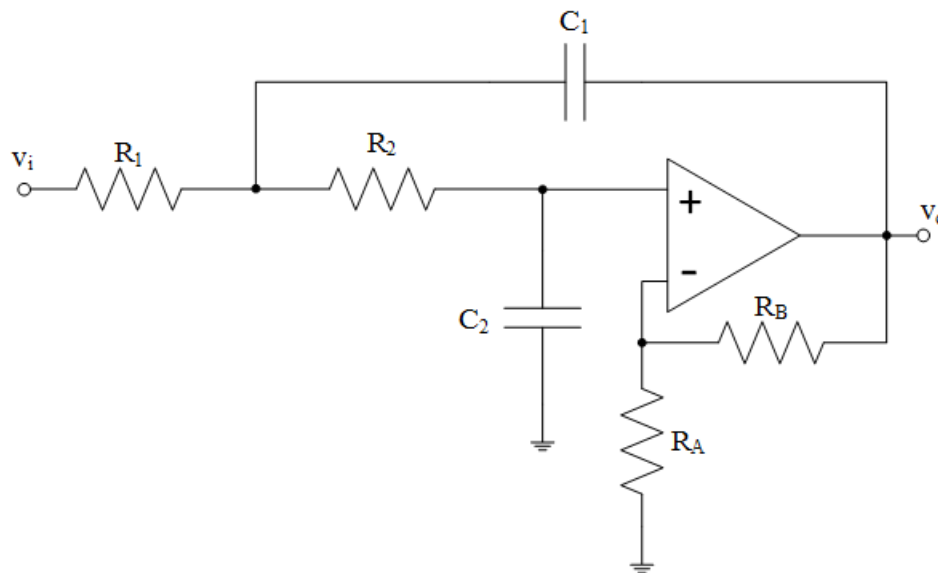


Figure 1: Sallen-Key Low-Pass Filter Circuit

Laboratory Procedure:

1. Construct the analog Butterworth filter designed in the prelab.
2. Perform AC analysis and verify that the filter specifications are met.
3. Plot the magnitude and phase response of the Butterworth filter.
4. Utilize your knowledge of Fourier series and the function generator to create a square or triangle wave in order to verify the filter's behavior.