EE 544 Homework 5

Consider the second order differential equation $\ddot{y}+2\zeta\,\omega_n\,\dot{y}+\omega_n^2\,y=K\,\omega_n^2 u$ with *y* the output, *u* the input, ω_n the undamped natural frequency, ζ the damping ratio, and *K* the DC gain.

1. Represent the system in both state-space form and as a transfer function.
2. Show that the poles/eigenvalues are $s_{1,2}=-\sigma\pm j\omega_d$ where $\sigma=\zeta\,\omega_n$ is the damping factor and $\omega_d=\omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency.
3. Solve for step response assuming an underdamped system, i.e., that $s_{1,2}$ are complex.
4. Show that the steady-state value of *y* is *K*.
5. Show that the time to the maximum (peak) is $T_p=\dfrac{\pi}{\omega_d}$ and that the overshoot is $M_p=K\,e^{-\zeta\pi/\sqrt{1-\zeta^2}}$.
6. Show that the 1% settling time is $T_s\approx\dfrac{4.6}{\sigma}$. What would the 5% settling time be in terms of σ?
7. Rise time T_r is a little more difficult to relate to poles/eigenvalues. Search around and see what you can find for a relationship (most often approximated as an average from range of responses). $T_r\approx\dfrac{1.8}{\omega_n}$ is one I've seen used.
8. Pick values, say $K=2,\omega_n=4,\zeta=0.4,$ such that response is underdamped and plot response (either numerically solve DE or directly plot y(t)). Confirm metrics by matching values on plot to calculated/expected values.